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No. 5

The Physics of the Developed Photographic Image

By F. E. Ross

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Preface to the Series

The Research Laboratory of the Eastman Kodak Company was founded in 1913 to carry out research on photography and on the processes of photographic manufacture

The scientific results obtained in the Laboratory are published in various scientific and technical journals, but the work on the theory of photography is of so general a nature and occupies so large a part of the field that it has been thought wise to prepare a series of monographs, of which this volume is the fifth In the course of the series it is hoped to cover the entire field of scientific photography, and thus to make available to the general public material which at the present time is distributed throughout a wide range of journals Each monograph is intended to be complete in itself and to cover not only the work done in the Laboratory, but also that available in the literature of the subject

Naturally a large portion of material in these monographs will be original work which has not been previously published, and all views expressed by authors of a monograph are not necessarily shared by other scientific workers in the Laboratory The monographs are written by qualified specialists who are given wide discretion in the expression of their opinions However, each monograph is edited by the Director of the Laboratory and by Miss Garvin, the active editor of the series

Rochester, New York
May, 1924

Preface

It is the aim of the present volume to present data on and concepts and interpretation of the behavior and properties of the photographic emulsion which are of interest primarily to astronomers and physicists. Accordingly, many matters of interest in the making of pictures, such as concern the amateur and professional photographer, and points of contact with the photographic arts, which might properly be classified under physics of photography, have been omitted.

The failure of the reciprocity law, an important branch of the subject, has been touched upon only lightly. A large amount of data in this field is now being accumulated in this Laboratory, which, combined with the fact that the whole subject is at the present time in an exceedingly unsatisfactory state, makes any extended presentation at the present time premature.

For similar reasons, on account of the limitation of the subject matter to the developed image, many matters of interest to physicists and astronomers connected with the properties of the undeveloped grains, and the formation of the latent image, upon which much of the physics of the developed image is necessarily based, are given only general treatment. The large amount of work at present being done in these excluded sections, in this Laboratory and elsewhere, taken in conjunction with its highly controversial nature, is considered sufficient apology for this omission. It was not possible to adhere rigidly to this rule, a certain overlap being necessary to properly develop the subject.

The historical development of the subject has been emphasized. An outline of the investigations and conclusions of other investigators in the field has been presented in such a way, and without any serious attempt at criticism, that the reader's concept of the subject as a whole may be connected and logical. Some of the data and conclusions of the writer which are contained in the present volume have been published in the *Astrophysical Journal* and in the *Journal of the Optical Society of America*. The second chapter, dealing with the subject of graininess, was written by A. C. Hardy, from the very extensive mass of data which he accumulated on the subject while a member of the Laboratory.

Rochester, New York
May, 1924

The Physics of the Developed Photographic Image

CONTENTS

The Physics of the Developed Photographic Image

CHAPTER I

The Developed Silver Grain

In any detailed study and description of the physical properties of the developed silver grain of the photographic plate, the following subdivisions suggest themselves:

1. Composition of the developed grain;
2. The relation of the developed grain to the latent image, including the subdivisions: (*a*) relation between the mass, size, and shape of the developed grain to the mass, size, and shape of the silver bromide grain in or adjacent to which the hypothetical latent image is supposed to reside; this relationship possibly extending to other adjacent grains which have not been acted upon by light but which by reason of their proximity are to be considered as possibly effective in the chemical process of building up the developed grain; (*b*) relation between the positions of developed and undeveloped grains.

The subject of the developed grain is naturally divided into two parts: (*a*) grains formed by physical development, such as occur in collodion and albumen emulsions; (*b*) grains formed by chemical development, occurring in the gelatin dry plate. The possibility of development in which both processes occur simultaneously must also be considered. On account of its greater importance and the wider range of phenomena, both physical and chemical, which it exhibits, the major portion of the present chapter will be devoted to the grain of the gelatin plate.

The first of a series of important investigations of the developed grain of the dry plate is due to A. and L. Lumière and Seyewetz.[1] Their conclusions are best summarized in their own words:

"The size of the grains of reduced silver deposited by normal developers, as ordinarily used, is practically invariable. No apparent influence is shown on the size of the grain of reduced silver by temperature, concentration, or duration of development. . . . Two developers not generally used, viz., paraphenylenediamine and orthoaminophenol, used with sodium sulphite, produce a deposit like that obtained when collodion emulsions are used, and of which the grain is much

[1] Lumière, A. and L., and Seyewetz, W., The influence of the character of developers on the size of grain of reduced silver. Brit. J. Phot. 51: 630. 1904.

ner than that produced by other developing substances" The peculiar action of these last two developers is explained y their "weak developing energy," combined with their bility to "dissolve appreciable quantities of silver bromide"[1]

We have here a case of combined chemical and physical evelopment in which the character of the grain is intermeiate in type The authors strengthen their explanation of finerainedness by showing that the addition to the normal evelopers of a solvent of silver bromide, as, for example, hloride of ammonia, the rate of development being at the ame time reduced by dilution or the addition of a suitable estrainer, is sufficient to produce fine grain

Practically coincident with the appearance of the papers of Lumière and Seyewetz appeared a paper on the same subject by R J Wallace[2] Wallace's results, in part, are in accord with those of Luppo-Cramer, who found that each particle of silver in the negative corresponds to a silver bromide grain n the undeveloped layer, and that the number of grains of silver in the upper layer of the emulsion does not increase appreciably with exposure or development, although the number per unit volume throughout the thickness of the emulsion does increase, not only in number but in size as well, for increased exposure and development In contradiction to the conclusion of Lumière and Seyewetz quoted above, Wallace finds with Seed 27 plates, appreciable dependence of grain-size on developers of the normal type, as the following table shows

TABLE 1

Developer	Grain Size in µ
Rodinal (1 hour's development)	3 0 to 8 7
Caustic potash and hydroquinone (6 minutes' development)	1 3 to 2 4
Caustic soda and hydroquinone and metol and adurol (1⅓ minutes' development)	1 4 to 1 8

The author remarks "In the case of the slow development by rodinal the character of the grain is vastly different from that of the remaining two plates, being decidedly more 'ragged' in appearance and showing an actual and definite increase in size, principally by reason of the running together of the several particles to form a new group particle In the case of the plate developed with hydroquinone the 'grain' is better, with less running together, while in that developed rapidly in the

[1] Lumiere A and L and Seyewetz W A process of photographic development for the production of images of fine grain Brit J Phot **51** 866 1904

[2] Wallace R J The silver grain in photography Astrophys J **20** 113 1904

hydro metol-adurol mixture the grains of silver are seen to be deposited in a much more definite and regular form than in either of the two preceding." Wallace watched development under the microscope, concerning which he remarked: "The grains of silver bromide when first acted upon by the developer were reduced as individual grains of very small dimensions, only a portion of the 2AgBr particles appearing to be acted upon at first, but increasing in size individually and then coalescing into group-particles which become larger as development is continued." Wallace comments on the apparent discrepancy between his results and those of Lumière and Seyewetz, which in his opinion is due to the latter diluting their gelatin before examining the grains embedded therein, a method which would tend to break up the group particles. Wallace considers it best to examine the grains *in situ* for the most reliable results.

Sheppard and Mees[1] made many investigations on the grain of gelatin emulsions which are summarized in their book. The authors by way of comparison and supplementation refer frequently to Bellach's monograph, *Die Struktur der photographischen Negative.*

According to Bellach, the top layer of a developed plate contains comparatively few grains, and the grains in the lower layer are of smaller size. The authors find this last statement to be true only for incomplete development. They find, in addition, that for short development the size of grain increases with the exposure. When the development is complete the grain-size is the same for all exposures.

Depth of Image. For full development, it was found that the thickness of the layer of developed silver increases with the exposure, but for incomplete development this was not the case, the depth of the layer being dependent merely on the degree of development.

Number of Grains. The authors find that, for full development, the number of grains in the surface layer does not increase with the exposure. When the number of grains throughout the entire thickness of the film was counted, it was found to increase with the exposure in direct proportion to the increase in density. This is not in agreement with Bellach. According to him the *number* of grains per unit volume is constant for all exposures, the observed increase in density in this case being explained as due to an increase in *size* of grain. The authors find a rapid increase in number of grains with development, apparently according to an exponential law.

[1] Sheppard, S. E., and Mees, C. E. K., Investigations on the theory of the photographic process. (Longmans, 1907.)

Sheppard and Mees remark that with respect to the theory of the time-dependence of development on depth, the time of diffusion of the developer to the lowest layer of the film is so small as to be without effect They accordingly conclude that if the action in the lower layers is slow, it must be due to greater concentration of the products of reaction which clog and slow the development process

From a comparison of results of exposures through the back of a plate with those made from the front, the important conclusion is drawn that those grains which have received the greatest exposure are developed first

They support the conclusions of Lumière and Seyewetz that normally the character of the developer is without influence on the grain size In the exceptional cases of fine-grain development "the reducer acts as a solvent for silver bromide It reduces the concentration of the silver ions, and at certain stages acts similarly to bromides" With high exposures a tendency to form clumps of grains is noted

The next important investigation was by W Scheffer,[1] who from a study of a large number of photomicrographs and sections of developing and developed plates, was led to novel and important conclusions The grains were studied at an enlargement of 2000 diameters The clearness and definiteness of the author's description of phenomena actually observed deserves full quotation Describing the first stages of the development process, Scheffer states "Jutting out from the grains are the shorter and longer filaments which are either straight or irregularly curved and mostly terminated in a knob These filaments have also at times thickenings in their length besides the terminal knobs The smaller grains have comparatively more of these filaments than the larger ones, if we take the relative masses into consideration It gives me a strong impression that the formations described here are the results of something similar to an explosion which takes place during the exposure Small bodies are shot away from the grains and make their way through the gelatin either in straight or in irregularly curved lines The filaments are formed by parts of these small bodies shot off which remain in the path Sometimes at the end of the path the small particles can be seen as a terminal knob In other cases the whole is used up on the way Both the terminal knobs and the filaments are the germs at which development commences Sometimes the filament is hardly visible even with the highest

[1] Scheffer W Microscopical researches on the size and distribution of plate grains Brit J Phot **54** 116, 271 1907

power oil-immersions." Fig. 1 is a reproduction of one of Scheffer's photomicrographs showing the grain and attached filaments. As the development proceeds "the fine germs are changed into more or less clumsy bodies which rest upon the original grains and have partly grown around these. . . These researches show that the germs at which the formation of the developed grain commences are situated outside the original

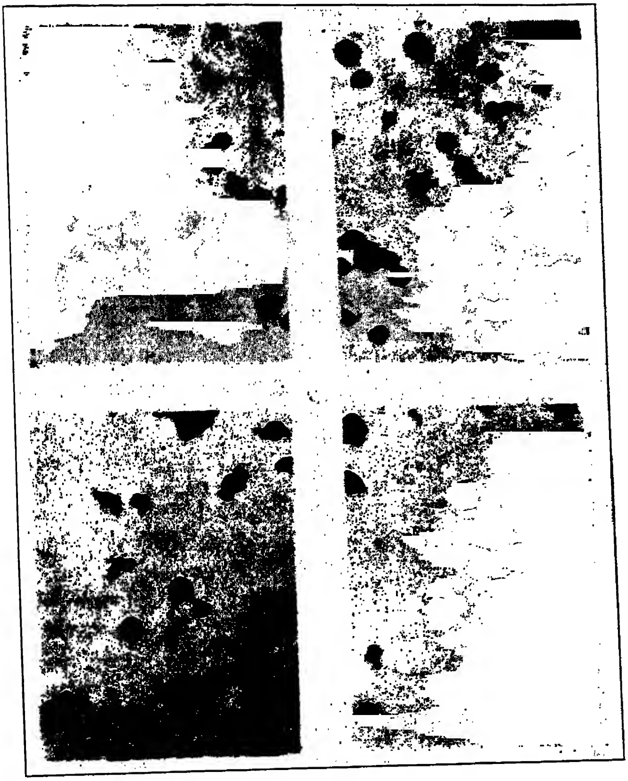


FIG. 1
Filament structure of developing grains, by Scheffer

grains and that also the further stages of development take place outside the original grain. . . . Only a certain part of the grains become original grains for the development. The others are dissolved by the developer. Examination of a great number of preparations shows that probably the number

of original grains and the number of those which take no part in the development are about equal in sufficiently exposed plates Even if we expose for a very long time it is not possible to make all the grains original grains Accordingly there are three classes of grains

(1) Original grains—grains which have germs around them, these germs being the points at which development commences These original grains are not dissolved by development

(2) Dissolving grains, grains which show no germs, and which dissolve either partly or entirely by chemical development

(3) Developed black grains "

In studying the effect of exposure on grain-size Scheffer concludes from a comparison of overexposure, normal exposure, and light exposure, that the developed grains are larger in size on the normal exposure than on the light exposure or on the overexposure From the relative number of dissolving (nourishing) grains visible in the three cases, he concludes that "the solubility of the dissolving grains in chemical developers is governed by the exposure, and the solubility increases at the commencement, corresponding with the exposure, up to a maximum, after which it decreases with increasing exposure " He further found that the point of maximum solubility was deferred with greater concentration of the developer, which means that solarization is a function of development In addition it was found that with greater concentration the maximum grain-size was reached at a weaker exposure A further conclusion of considerable importance was that "the size of the developed grains depends on the number of grains in the unit volume of gelatin The less grains suspended in unit volume, the smaller is the size of the developed grains "

In a paper several years later, Scheffer[1] goes further into the subject as follows "From experiments which I have recently published on the microscopical examination of the grains of the plate, it follows that neither the size nor the position (locus) of the black developed grain corresponds exactly with the size and position, respectively, of the light impression The position of the first traces of development, the so-called germ, always lies outside of the exposed silver particles, the 'original grains,' as one may call them, and the black developed grains grow around this germ The direction of growth is not dependent on the place (locus) where the light

[1] Scheffer, W On the resolving power of photographic plates Brit J Phot 57 24 1910

acts This place or position of the black developed grain corresponds only within certain limits with the place of the light action The smaller the black developed grain the nearer it lies to the 'original grain,' and the smaller this is with corresponding closer approach will the position of the black developed grains correspond with that of the light action The size of the black developed grain is distinctly dependent on the development (and upon any after-treatment), much more so than upon exposure With increasing exposures the number, but not the size, of the black developed grains increases" It is to be noted that the last statement in this quotation is not in accord with the author's earlier paper quoted above

The conclusion of C W Piper,[1] while of primary interest in connection with theories of the latent image, is nevertheless of importance in the present connection in that the subject of the position of the latent image with respect to the developed image is discussed He says "First we must admit that the latent image, whether produced by light or chemical action, is itself undevelopable, but that in its presence the immediately adjoining silver bromide becomes developable, this being quite opposed to the general idea that the image as a whole is developable because the latent image is developable Secondly, we must consider reversal to be wrongly so-called, and must look upon it as the simple consequence of the continuing action of light, or of the production of an excess of undevelopable product Thirdly, we may look upon development as being an action of a catalytic nature, the latent image being the catalyzer" The similarity of the views of Piper and Scheffer, at least in their broad outlines, is to be noted

Some very rough measurements have been given of the size of the grain of a high speed plate The average size of the grain and the range of variation in the size in any given emulsion vary enormously, depending on the character of the emulsion It is an easy matter to determine the upper limit of grain size in any emulsion To determine the lower limit is not so easy, values obtained depending on the optical power employed Working under the very best conditions it appears to be possible to measure directly grains of 0 2μ in diameter Chapman-Jones,[2] by an indirect method, has measured grains of about one-half this size and in so doing has been able to establish a connection between the color of the plate and the average grain size His method consists in enlarging the

[1] Piper C W The nature of photographic images Brit J Phot **55** 195 1908

[2] Chapman-Jones On the relationship between the size of the particle and the color of the image Brit J Phot **58** 381 1911

grains by intensifying a given number of times From measurement of the enlarged grain and calculations based on the chemistry of enlargement, he was able to compute the size of the original grains Correlating the measured sizes with the colors exhibited by the emulsion by transmitted light, made it possible to draw important conclusions as follows a plate that looks yellow by transmitted light is composed of grains averaging 0 12μ in diameter For increase in diameter the color changes continuously until when the particles are 0 30μ the color becomes gray He concludes that "particles that have a diameter equal to about a quarter of a wave-length produce very little, if any, effect on that wave, but that as soon as they bear a substantial proportion to the half wave-length the effect becomes considerable"

The composition of the developed image has been determined by A and L Lumière and Seyewetz [1] They find that the silver of the developed and fixed image is not pure silver, but contains sulphur and iodine, in the case of brom-iodide plates This explains why the developed image is not completely reduced by many solvents of pure silver, for example, by potassium permanganate The same fact was discovered by Carnegie[2] some years previously

The next investigation in point of time on the subject of the silver grain was by Tugman [3] Novelties in Tugman's investigations were the use of very thick films, sectioning the films before development, and in addition, a series of exposures were made to X-rays, thus allowing the various factors in the problem to be well differentiated The primary object of the investigation was determination of the distribution of the grains downward in the film To determine this two methods were employed (*a*) counting the number of grains, (*b*) measurement of density with a microphotometer Both classes of measurements agreed in showing a diminution in grain frequency downward in the emulsion according to an exponential law As remarked above, all measurements were on sectioned strips of film, sectioned before development, so that dependence of development on depth was effectively eliminated Interesting results on the penetration of light of various frequencies were obtained It was found that for an exposure to ultra-violet light, the maximum penetration obtainable was 30μ With the particular intensity used no increase o

[1] Lumiere A and L and Seyewetz A The composition of the developed image Br J Phot 59 61 1912

[2] Carnegie D The chemistry of the sulphide toning process B J Phot 53 947 190

[3] Tugman O The distribution of the silver grain in the developed photographic imag Phot J 38 270 1914

exposure time caused this limit to be exceeded Since the average thickness of commercial emulsion is from 10μ to 20μ, it follows that light of all wave-lengths is able to reach the back of the ordinary plate For blue light the maximum depth reached was 70μ, or more than double that of ultra-violet

To show the limits of penetration of developer, a thick film was exposed to X-rays, stripped from the plate, and developed without sectioning The result is shown in Fig 2 Surface

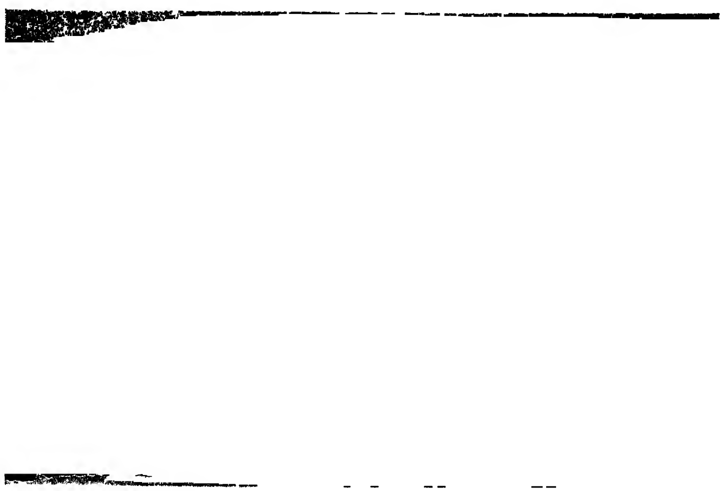


FIG 2
Limiting penetration of developer

layers only 12μ thick are seen to be developed, which accordingly, is the maximum penetration of the particular developer used A further interesting result was obtained by triply coating a plate, each layer being allowed to set before the succeeding layer was added Exposing, sectioning, then developing, and counting the grains, a periodicity in grain frequency was shown, the grains being relatively more numerous near the surface of each layer.[1]

Nutting[2] has briefly described a number of the characteristics of the developed silver grain, as follows "The grains are not crystals, but aggregates of finely divided silver resembling platinum-black or soot of very high absorbing, and low

[1] This suggests a greater sensitivity at the surface of an emulsion due possibly to quicker drying

[2] Nutting P G, On the absorption of light in heterogeneous media Phil Mag 26 423 1913

reflecting power This reflecting power has not yet been directly determined but estimates based on scattering make it well under 2 per cent "

A conception of a portion of the physical mechanism of development has been given by Mees[1] which is of interest in the present connection since it involves the structure and properties of the gelatin in which the grains are embedded "We may consider an emulsion to consist of crystalline granules of silver bromide inhabiting cells of a jelly which, on setting, surrounds each grain of the silver salt with a wall of gelatin Of the structure of the grain itself very little is known In high speed emulsions it is probable that the grains consist of crystals, while in low speed emulsions the grains themselves may be partly composed of gelatin, separating still finer crystals of silver bromide Through the channels in the gelatin, ions and molecules can travel with ease at a velocity comparable with their diffusion rate in water, while penetration through the cell walls is a much slower and more difficult process Diffusion into gelatin and especially the diffusion of the developing solution into the emulsion film may, therefore, be divided into two processes

(*a*) Macro-diffusion, or the penetration of the solution through the channels of the gelatin, and

(*b*) Micro-diffusion, or the penetration through the walls, which is usually necessary to some extent before the developer can reach the silver halide grains "

Some experimental work on grains has been undertaken by M B Hodgson[1] principally along the general lines outlined by Scheffer, in the hope of observing the phenomena which he so vividly described The progress of development was followed with a high power microscope and photomicrographs were made The phenomenon observed by Scheffer of filaments shooting out from the developing grain was obtained only in one exceptional case of strong overexposure on a thin film, which the writer attributes to a crack in the gelatin Instead of observing the grain developing along filaments as Scheffer found, development was found to proceed from "spots" on the grain itself, the developing areas expanding until the entire grain was developed These spots are shown well in Fig 3, from a photomicrograph secured by Hodgson In general, he notes that "During development the growing silver

[1] Mees C E K The physics of the photographic process J Frankl Inst **179** 141 1915

[1] Hodgson M B, Physical characteristics of the elementary grains of a photographic plate Brit J Phot **64** 532 1917

mass becomes twisted and distorted. . . . . The developed grains are masses of fine, sooty, black silver, held together by the surrounding gelatin. They may be separated by swelling the gelatin or by applying other mechanical force." Fig. 3 is from a photomicrograph of a grain near its initial stage of development and at its completion, the crystalline forms being preerved. The observed increase in size is

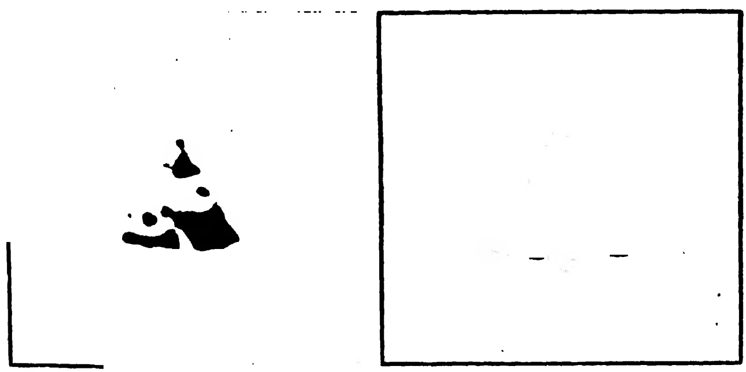


FIG. 3
Two stages of development, by Hodgson.

attributed by the author to an inclusion of gelatin during the development process. Experiments were made to determine if the silver bromide grain contained gelatin. Since no change in size of a silver bromide grain was observed under the microscope after applying water, it was concluded that no gelatin was present. Measurements were made of the average size as well as the variation in size of grains of various types of emulsions, with results shown in Table 2.

TABLE 2

Plate	Range in Grain Size in μ
Cine Positive Film	0.2 – 2.0
Seed 23 Plate	0.2 – 3.5
Seed Graflex Plate	0.2 – 6.0
Special Experimental Emulsion	0.2 – 8.5

The size of the grain of the wet collodion plate is the subject of an investigation by M. Monpillard.[1] Large differences in the size of the grain were obtained, depending on the developer.

[1] Monpillard, M., Experiments on the grain of silver images obtained in the wet collodion process. Brit. J. Phot. **51**: 866. 1904.

Comparing development with iron and with pyro, the size of grain in the latter case was but one-half that obtained with iron The average grain-size of a collodion emulsion was found to vary within wide limits according to the type of collodion and developer In some cases the average grain-size was 0 4μ, in others it was as great as 2 5μ, the range being similar to that in dry plates

CHAPTER II

Graininess

While the scientific investigator has been devoting his attention to the minute silver grains which constitute the photographic deposit, the photographer has been interested in a somewhat related but essentially different property of the photographic material. To observe the ultimate silver particles in the deposit, the scientist uses the most powerful microscope at his disposal. The photographer, on the other hand, is interested in the appearance of the photographic deposit when it has been only moderately enlarged as in the ordinary bromide enlargement or on the motion picture screen. Under such conditions, it is impossible to see the individual grains, although the clumping together of the grains may cause a lack of homogeneity which may be very apparent. To this type of non-homogeneity, the term "graininess" is applied in order to distinguish it clearly from the effect produced by the resolution of the individual grains under much higher magnification. While it is undoubtedly true that the graininess of a given photographic deposit is some function of the size-frequency and distribution of the individual grains which constitute it, the functional relationship has not been established. At the present time it is necessary to measure graininess by more direct methods which do not involve measurements of the individual silver grains.

A satisfactory method of measuring graininess has been described by Jones and Deisch.[1] It is based on the fact known to the user of photographic materials that there is a definite limit to the degree to which a photographic image can be enlarged before any lack of homogeneity becomes apparent. That is, if a number of enlargements are made from the same negative with varying degrees of magnification, it will be found that up to a certain size the graininess will not be apparent. At one particular degree of enlargement, the graininess will be noticed and for higher magnifications it becomes more noticeable. It is, of course, impracticable to measure the graininess of a photographic material by making a series of enlargements and selecting the one which first shows any indication of the granular structure. However, the equivalent procedure can be carried out by means of the instrument described by Jones and Deisch.

[1] Jones, L. A., and Deisch, N., The measurement of graininess in photographic deposits. J. Frankl. Inst. **190**: 657. 1920.

The instrument consists of a microscope system which projects a magnified image of the photographic deposit onto a screen of magnesium carbonate The observer places himself in such a position that he can see the screen in a mirror attached to a carriage which operates along a track By moving the mirror, the total distance from the eye of the observer to the screen may be varied at will The assumption is made that the graininess of a given material is proportional to the distance at which the appearance of graininess becomes just perceptible, provided that all the factors upon which depend the ability of the eye to distinguish lack of homogeneity are constant It will be seen that measuring the distance at which the graininess becomes apparent in an image of constant magnification is equivalent to measuring the magnification at which the graininess becomes apparent with a constant distance of observation

A diagram of the essential parts of the instrument constructed by Jones and Deisch is shown in Fig 4 Light from a Pointolite lamp (S_1) passes through the sub-stage condenser

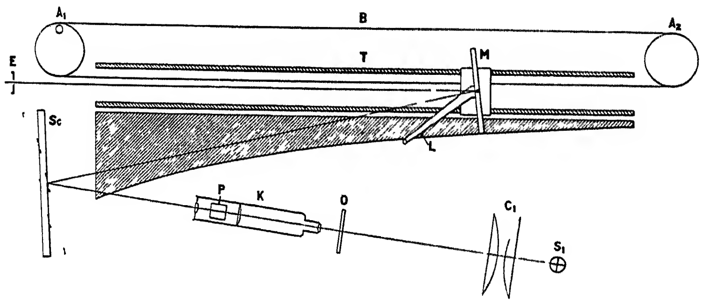


FIG 4
Instrument for measuring graininess

(C_1) and the photographic material (O) placed on the stage The microscope (K) with prism (P) projects a magnified image on the magnesium carbonate screen (Sc) The observer, by placing his eye at the eyepiece (E) sees the image of the screen which is reflected in the mirror (M) This mirror operates along the track (T) and is controlled by the template (L) which maintains the mirror in such a position that the screen is always visible to the observer The carriage carrying the mirror is driven forward and backward by means of the tape (B) which is provided with a scale indicating the distance of the mirror from the observer From the reading of the scale,

the total distance from the eye of the observer to the image of the screen seen in the mirror can be calculated. In the instrument described, the track was of sufficient length to permit this distance to be varied from two feet to more than sixteen feet.

Assuming the visual acuity of the eye of the observer to remain constant, the procedure in measuring graininess with this apparatus would be to place the photographic material on the stage of the microscope and drive the mirror out slowly until the screen appeared to blend into a smooth homogeneous area. The scale would then be read at this point where the sensation of granularity first vanished. The total distance from the eye to the screen for this position would then be a direct measure of the graininess of the material. Thus, it could be said that a material with a blending distance of six feet was twice as grainy as one which had a blending distance of only three feet.

It is well known, however, that the visual acuity is continuously changing and depends upon a number of factors, the principal ones being the adaptation level of the eye, the contrast and color of the field of view, the general physiological condition of the observer and such other uncertain conditions as fatigue and practice. To eliminate the effect of changes in the visual acuity on the graininess measurements, the following scheme was adopted. In computing the graininess of a given photographic material, the blending distance for the material was divided by the corresponding distance determined for a printer's half-tone screen. This ratio of the two distances was then used as a measure of the graininess of the photographic material. The same half-tone screen was used throughout and in this way, the ratio of the two resolving distances was found to remain constant, although both distances might vary considerably from day to day. In fact, it was even found possible for two observers to agree on the graininess of a certain photographic material when the resolving distance of the material obtained by the one observer might be very different from that determined by the other.

Several improvements were later made in the technique of measuring graininess which are described in a paper by Hardy and Jones.[1] As a result of these modifications, it was found possible to measure the graininess of a photographic material with such precision that the average deviation of the graininess factor for a single observation would not exceed three per

[1] Hardy, A. C., and Jones, L. A., Graininess of photographic materials used in the motion picture industry. Trans. Soc. Mot. Pict. Eng. No. 14, p. 107. 1922.

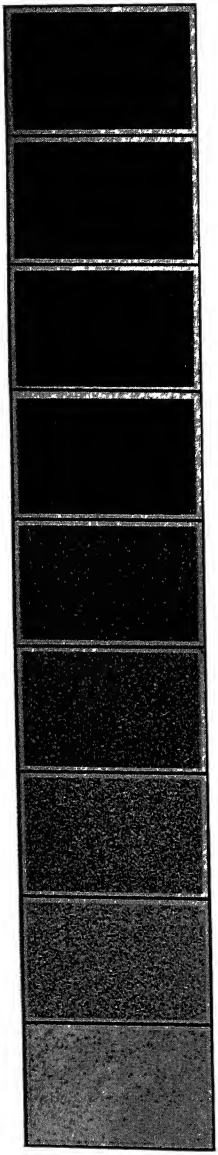


FIG 5
Enlargements showing relation between density and graininess

cent under ordinary conditions Considering the errors which usually occur in the measurement of the constants of photographic materials, this precision was considered satisfactory

In the paper referred to above, the authors show that the graininess values measured with the instrument correspond to the graininess as already defined and as understood by the photographer To demonstrate this, a typical negative material was selected and given a graded series of exposures in a sensitometer and developed in the usual way The result was a negative consisting of a series of densities, each density having, of course, its own graininess factor Since the graininess of the material was too small to permit even the larger clumps of grains to be seen with the naked eye, photomicrographs were made of each step, giving to each step the same exposure and time of development regardless of its density Prints were then made of each giving the same exposure and development throughout A photograph of these prints after they had been mounted together in the proper order is shown in Fig 5

Since the photomicrographs were made under identical conditions of exposure and development, this figure represents the appearance of the original negative when magnified nearly one hundred diameters This makes it possible to judge the graininess which would otherwise be difficult to reproduce It will be seen that the maximum graininess occurs in the neighborhood of the third step This can easily be verified by placing this page at such a distance from the eye that it is only possible to resolve the grains in one of the steps The figure shows also that the graininess diminishes on either side of the maximum as would be anticipated

It is unthinkable that a photographic deposit of zero density or one of infinite density could have any apparent granular structure.

This negative was then placed in the graininess apparatus and the graininess factor[1] of each step determined in the usual way. The values of the graininess factor for each step plotted as a function of the corresponding density are shown in Fig. 6. It is evident that the curve described the graininess of the

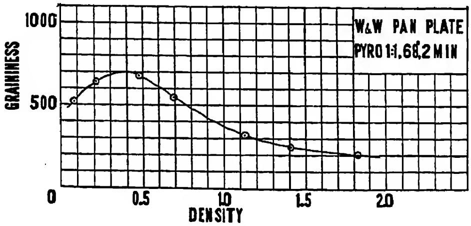


FIG. 6
Graininess-density curve.

material exactly. The maximum graininess occurs in the neighborhood of the third step and the graininess falls off on either side of the maximum as before.

A little consideration will show that there is reason to expect that the maximum graininess should occur in the neighborhood of an optical density of 0.3. The photographic deposit is composed of grains of black metallic silver which have a very low reflecting power and are almost opaque. A photographic density is therefore some configuration of alternate light and dark areas. When the light areas (the spaces between the grains) predominate, the optical density of the deposit, and also the graininess, approach zero. With this concept of the photographic deposit, it is to be expected that the maximum graininess would occur in the neighborhood of an optical density of 0.3 or a transmission of one-half. In this case the total area occupied by the grains themselves is equal to the total area of the spaces between the grains. It is known that when a number of uniformly spaced parallel lines are ruled in black ink on a white card, the lines can be

[1] The graininess factor is the product of an arbitrary constant and the ratio of the merging distance of the negative to the merging distance of the half-tone screen. Thus, if K is the arbitrary constant, D_1, the merging distance for the photographic material and D_2, the corresponding distance for the half-tone screen, the graininess factor, G, is expressed by:

$$G = K\frac{D_1}{D_2}.$$

The arbitrary value of K chosen was 508 and the half-tone screen had 500 lines per inch. The details of the other conditions are given in the original paper.

seen at the greatest distance when the width of the line is equal to the white space between them For somewhat similar reasons, the maximum graininess should occur when the total grain area is equal to the total area of the interstices Hardy and Jones found that there were other factors which at times constrain the maximum graininess to occur at a slightly different value of density but agreement with expectation is good enough to constitute a demonstration of the validity of the method of measuring graininess

Hardy and Jones first turned their attention to the graininess of photographic plates in common use in astronomy and spectroscopy In such work, all measurements are usually made directly on the negative and a print is made only occasionally for purposes of reproduction Consequently, it is the graininess of the negative material which is of interest and the procedure adopted by the investigators is as follows The plate to be tested was placed in a sensitometer and given a graded series of exposures The exposures were increased geometrically by the cube root of two, that is, the exposure was doubled every three steps In this way it was possible to get a sufficient number of points to determine the shape of the curves very accurately

After exposure the plate was cut into strips about an inch wide and four inches long and developed under various conditions Considerable care had to be used to keep the plates free from grit and dirt of all kinds which would be recorded later as graininess Other details of the procedure during developing, fixing, washing, and drying of the strips will be found in the original articles When dry, the optical density of each step was measured and the strip then placed in the graininess apparatus and the graininess factor of each step determined as already described The results were plotted showing the graininess factor as a function of the density of the deposit for various photographic procedures

Typical curves are shown in Fig 7 for a number of plates which are commonly used in astronomy and spectroscopy In this case each plate was developed in a caustic-hydroquinone developer for the same length of time The length of time (3 minutes at 63° F) was sufficient to give a high contrast, which is usually desired in plates intended for this purpose It will be noticed that the maximum graininess occurs in the same vicinity of the same density value for all the materials Further, the maximum graininess increases with the speed of the plate, that is, the maximum graininess of a Seed 30 plate is less than a Seed Graflex plate which is a slightly faster plate

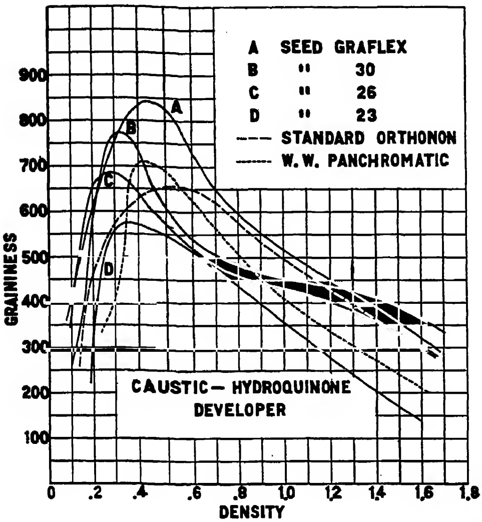


Fig. 7
Graininess of a number of emulsions.

In the same way, the maximum graininess of a Seed 26 is less than a Seed 30, and a Seed 23 plate less than a Seed 26. The orthochromatic and panchromatic plates, represented by the Standard Orthonon and Wratten and Wainwright Panchromatic plates, belong to a different series and have slightly different forms of curves with a maximum which falls intermediate between the Seed Graflex and the Seed 23.

The interpretation of the graininess-density curves when plates are to be used for astronomical purposes must be slightly different from the ordinary case. It is customary to judge the graininess of photographic materials in practice by a visual examination of a sensitometric strip. In this case, the eye is attracted to the step whose grains are the most in evidence. Thus, the eye quite naturally finds the step of maximum graininess and compares two different photographic materials at their maximum density. However, in astronomy and spectroscopy, the densities corresponding to the maximum graininess are rarely obtained. For example, a photograph made of the night sky through a telescope might contain a large number of star images, the density of which would probably be high. The remainder of the plate would receive no exposure except for the scattered light in the apparatus. It is usually found that the latter is sufficient to give a density which, although considerably less than the density corresponding to the maximum graininess, is sufficient to make the graininess of this area of much more importance than that of the star images. The graininess in the case of star images is noticed at the edge of the image where, under examination with a low power microscope, it prevents an accurate determination of the exact position of the edge.

Hardy and Jones determined the graininess-density curves for these same materials with different photographic procedures They found that, with only a few exceptions the graininess of the plate did not depend upon the nature of the developer nor upon its concentration Most of the ordinary developing agents such as pyro, metol and hydroquinone, in various combination, amidol, etc , were tried, but no regular difference was found between them Such developers as paraphenylenediamine were found to give less grain but also a low maximum contrast (gamma infinity) which makes them of little use for such work

The effect of developing the plate to different contrasts (gammas) was investigated with the ordinary developers and a slightly lower graininess was found with a very low contrast The graininess seems to increase very rapidly with the contrast to a fairly constant value which holds for most of the useful contrast range of the plate It appears from these results that there is little which can be done to diminish the graininess of plates used for astronomy or spectroscopy The use of a slower plate in general gives a lower density and any means of reducing the amount of stray light falling on the plate will also decrease the effective graininess Care should likewise be taken to prevent chemical fog as much as possible

With the exception of astronomical and spectroscopic uses, the photographic process is usually not complete until a positive print has been made This adds a new variable to the problem in the graininess of the positive material It was previously assumed[1] that such positive materials as developing-out papers and cine positive film had an intrinsic graininess so much lower than that of the negative materials that the graininess of the positive itself could be neglected This is true in the sense that film would have a graininess of a very different order of magnitude from the same photograph on cine positive film Later work has shown that the graininess of the positive material is nevertheless of considerable importance in determining the graininess of a positive when printed from a negative This can be demonstrated very easily in the following manner without recourse to an instrument for measuring graininess Take a sensitometric strip made on a typical negative material and make from it a positive print on cine positive film or a lantern slide plate Then make from this print a duplicate negative using again cine positive film or a lantern slide plate If care is used in the exposure and development of the latter it will be possible to duplicate almost

[1] Jones L A and Deisch N l c p 681

exactly the densities of the steps of the original negative. If the duplicate negative is then compared with the original it will be found that the graininess of the duplicate has been enormously increased. Now, if the positive material had no influence on the graininess of the print, it would have been possible to make a duplicate negative which would have the identical graininess of the original. In other words, the pattern of the grains of the original would have been reproduced exactly in the duplicate.

It has been stated by Hardy and Jones that the increase in graininess during the printing process is due to the insufficient photographic resolving power of the positive material. The negative emulsion contains grains which vary in size between rather wide limits. Although the resolving power of the positive is sufficient to resolve the larger grains, it is insufficient to permit resolution of smaller grains in the emulsion which, therefore, do not appear in the print. The result is that the areas between the large grains become uniform dark patches in the print many times larger than even the largest of the original grains. It will be remembered that the resolving power of the positive material is dependent upon a number of factors (Chapter IV, p. 143), one of which is the intrinsic graininess of the positive emulsion. However, the presence of other factors makes it impossible to predict the graininess of a print from the intrinsic graininess of the negative and positive material.

As might be expected from this, the authors found it necessary when investigating the graininess of photographic materials intended for ordinary uses, to measure the graininess in a positive made under the same conditions as would probably obtain when in use. They devoted their attention particularly to the graininess of motion picture positives. In this work, the magnification may be as high as two hundred diameters in a large theater and the graininess becomes a serious problem. Indeed, if it were possible to stop the projection machine without injury to the film, it would be found that a single picture is exceedingly grainy. Owing to the fact that one picture is followed by another in rapid succession, the graininess is less objectionable than in a single picture and appears as a boiling or scintillation of certain areas of the picture. This appearance of boiling is due to the slight differences in position of the grains in the different pictures.

The method adopted by these authors in attacking the problem was to expose the negative material (cine negative film) in the sensitometer and to develop it under standardized

conditions. The negative was then printed on the positive cine film, by contact in a printing frame which insured actual contact of the two films. The color and intensity of the printing light were controlled and the exposure regulated by varying the time. In making a print which is to be projected on the motion picture screen it is necessary to observe two conditions. First, the minimum density of the positive (the highlight) must be low. In other words, neither chemical fog nor overexposure can be tolerated since it decreases the intensity of the light on the screen. Secondly, the highest density (the deepest shadow) must not exceed 1.8 or 2.0 density units, since the amount of light which reaches the screen after passing through a higher density than this is insufficient with the present projection machine.

Hardy and Jones assumed that the average subject of which photographs are made has a contrast scale of one to thirty-two. That is, they assumed that the light reflected from the highlight of the picture area was thirty-two times as intense as the light reflected from the deepest shadow. They then selected on the negative sensitometric strip the number of steps which corresponded to this range of contrast and printed and developed the positive so that the highlight would have a density just slightly higher than the fog and the deepest shadow would be about 1.8. This procedure corresponds closely to the conditions under which the motion picture film is used.

They first investigated the effect of the exposure of the original negative. This was done by selecting in the printing process the steps of the sensitometric strip which corresponded to different exposures of the negative. The minimum exposure which can be given to a negative is the one which places the density of the deepest shadow of the subject on the toe of the characteristic curve near the beginning of the straight line portion. From this minimum, the exposure can be increased until the highlight of the picture is recorded on the shoulder of the characteristic curve. With a contrast range in the imaginary subject of one to thirty-two, it is possible to give an exposure of eight or more times the minimum and still obtain correct tone reproduction.

When prints were made according to this plan and measured in the apparatus for measuring graininess it was found that the longer exposure tended to increase the graininess. The best exposure to give the negative was found to be the one in which the deepest shadow received about half as much light as required to place the corresponding density on the straight

line portion of the curve. This means that part of the toe of the curve is used but this was shown to slightly improve the rendering of tone values. The increase in graininess with increase of exposure was very marked and was found in every case investigated.

The effect of the time of development of the negative was investigated. A number of pieces of film were exposed alike in the sensitometer and then developed for different lengths of time in the same developing solution. When identical prints were made from these negatives and measured on the graininess apparatus, it was found that the differences in the graininess of corresponding steps were too small to be measured. This means that it makes little difference whether the negative is developed to a high contrast and the positive purposely developed to a proportionately lower contrast or whether the opposite conditions obtain.

The same apparent contrast is obtained in the print when the product of the contrast factor (gamma) of the negative and the contrast factor of the positive is a constant. The writers found that, within the limits for which this relation is true, the graininess of the resulting print is independent of the procedure followed. Typical curves are shown in Fig. 8.

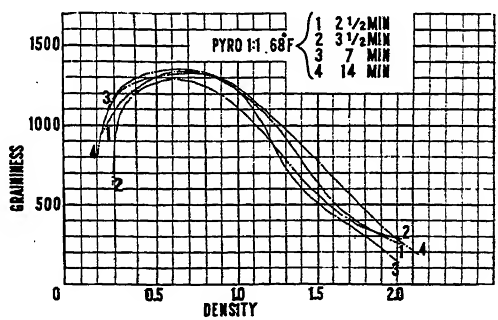


FIG. 8
Dependence of graininess upon development time.

It will be seen that the curves differ by less than the probable error of the determination of graininess and certainly by less than the least perceptible difference in graininess under ordinary conditions.

The effect of the concentration of the developer in which the negative was developed was then investigated. The general conclusion reached was that the graininess was increased by

dilution of the developer In this case samples of cine negative material were given identical exposures in the sensitometer and then developed in a pyro developer of the same composition but different amounts of dilution The time of development was so regulated that the degree of contrast (gamma) of each negative was the same Prints were then made under identical conditions and the graininess measurements made, as shown in Fig 9

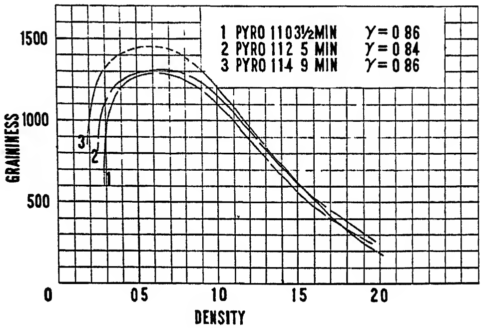


FIG 9
Dependence of graininess upon concentration of developer

A number of the common developing agents were tried to see if the development of the negative in any one of them tended to produce less graininess than any of the others The general conclusion was that the developing agent had only a very slight effect on the graininess It was found that a concentrated pyro developer gave the minimum of graininess and that the Eastman process developer, which is used to obtain extreme contrast, gave the most Between these limits which are not widely separated came the various compositions of metol and hydroquinone, amidol, etc Only the standard developers were investigated, as it had been found that such developing agents as paraphenylenediamine, which was known to give less grain in the negative, were not satisfactory for the development of cine film from which a print was to be made The limitations of correct tone reproduction prevented its use except in the case of subjects of unusual contrast

These authors found that the printing light had an effect on the graininess of the resulting positive They found that it made considerable difference whether a specular or diffuse beam was used, the lower graininess factor occurring with the diffuse beam This effect is probably due to the sharp shad-

ows which the grains of the negative cast by specular light, which are somewhat softened by diffuse light. The decrease in graininess is unfortunately accompanied by a sacrifice of definition which becomes very serious if the positive is not held in absolute contact with the negative during printing. For this reason most commercial printing machines are designed to give a nearly specular beam.

It may be well to add at this point that the exposure of the positive has an effect on its graininess in the following way. Normally, the exposure is determined by the density of the highlight portion of the negative, time being given to yield a just perceptible density in the developed positive.. However, in certain classes of subjects in which large areas of the picture would be reproduced by this procedure as densities very near the maximum of the graininess-density curve, it is well to alter slightly the exposure of the print. Decreasing the exposure will make all the densities lower with a sacrifice of correct reproduction of tone values in the highlights. Increasing the exposure, on the other hand, reduces the screen brightness by the same amount for each density and may cause a loss of detail in the shadows unless the current through the arc of the projection machine is increased proportionately. One class of subject, which usually exhibits excessive graininess, is an open seascape with clouds in the sky. The relatively large closed areas often appear to be boiling violently caused by the fact that the density is probably near the maximum of the graininess curve. In such a case, the exposure should be altered to keep the large areas of the picture of such densities that the graininess will not be noticed.

The color of the printing light has also an effect on the graininess of the print. The general relation found is that the light which gives the highest photographic resolving power gives also the minimum graininess. In the visible region of the spectrum the blue light is known to give the highest resolving power and is found to give also the minimum graininess. The exception seems to come in the ultra-violet where the resolving power is thought to fall off while the graininess is very much reduced. The resolving power of the photographic materials has not been investigated thoroughly in this region and it is possible that it will be found to increase when further work is done on the subject. At any rate, the exposure of the print by ultra-violet light such as obtained with a mercury lamp and a Wood's filter results in a marked decrease in graininess.

The effect of the developing conditions used when making the positive was investigated by selecting a typical negative and making from it a number of prints by different photographic procedures It was again found that the more dilute developer tended to give a slight increase in graininess This held true for most developing agents For the same concentration, the different reducing agents such as metol, metol-hydroquinone, amidol, etc , produced the same graininess within the precision of the method of measurement The Eastman process developer which before had given the highest graininess factor, this time produced the lowest graininess factor when that developer is used in developing the print This may be due to the increase in resolving power which results from the use of the process developer However, the alteration in graininess produced by the different developers is hardly sufficient to make any one of the common developers preferable to any other for this reason alone

CHAPTER III

Astronomical Photographic Photometry

While it may be affirmed that a study of photographic photometry in general involves a study of the structure of the photographic image to a certain extent, it is in a larger measure true when the special field of astronomical photographic photometry is to be investigated. This is due primarily to the wider range of the methods of astronomical photometry, and to the special circumstance that only very small images are utilized, requiring therefore a special study of their structure and mode of formation. Astronomical photographic photometry, accordingly, comes within the scope of the present monograph.

The subject of astronomical photographic photometry may be divided into two subdivisions, apparently unrelated, depending upon the following properties of the photographic plate: (*a*) increase of specific blackening or density with increasing intensity of illumination or increasing exposure time; (*b*) increase in size of an image with increasing intensity or exposure time. The range of measurement within which (*a*) is sensitive is comparatively small, since the blackening or density very soon reaches a limit, depending upon the thickness of the emulsion and the amount of silver contained in it, upon limitations of development, and upon other factors. In addition, there is the further difficulty of measuring high densities. These difficulties and limitations do not occur under (*b*), as will be seen.

Sensitivity. The usefulness of any photometric method depends primarily upon its sensitivity which mathematically can be defined as the ratio of effect to stimulus. In the case of method (*a*) above, the sensitivity is the ratio of the increase in density to the logarithm of the exposure; in (*b*) it is the ratio of the increase in diameter to the logarithm of the exposure. The ratio of the two is the relative value of the two methods in so far as sensitivity is concerned. It is not the purpose here to discuss this subject for it depends upon so many factors that it would take us too far afield. In general it can be said that method (*a*) is the more sensitive of the two, but that its intrinsic advantages are very greatly dissipated by apparent non-uniformity in sensitivity and development of the film. While it is theoretically possible to measure light intensities by the photographic method with an uncertainty of about one per cent, the errors actually obtained are many

times this amount Stetson[1] finds that the probable error of a magnitude determination from a single image by means of his very accurate thermoelectric method amounts to 05 to 08 magnitude To transform this to percentage intensity error, we have, calling I intensity, and m stellar magnitude,

$$m = m_0 - 2\ 5 \log_{10} I,$$

from which by differentiation,

$$-\frac{\Delta I}{I} = \frac{0\ 4}{Mod}\Delta m = 0\ 92\ \Delta m$$

Accordingly, Stetson's results mean a probable error in the determination of a light intensity varying from 4 6 to 7 4 per cent This means that an extreme range of about 40 per cent is to be expected in an extended series of measures of photographic light intensities By refinements in method, Stetson[2] has since been able to reduce the above probable error to ± 022 magnitude

There are important and interesting differences in the sensitivity characteristics of the two methods, which are shown graphically in Fig 10, curves A and B With method

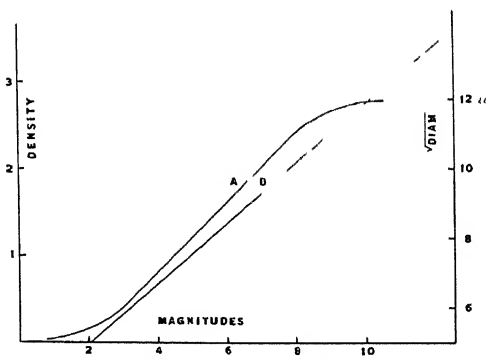


FIG 10

Characteristics of two photometric methods

(*a*), shown graphically by curve A, the sensitivity in the lower densities (toe) and in the higher densities (shoulder) is seen to be very low Only the intermediate portion of the curve or

[1] Stetson H T Preliminary note on the uniformity of film sensitivity of photographic plates from measurements with the thermoelectric photometer Pop Astronomy **27** 151 1919, The investigation of plate errors in photographic photometry Pop Astronomy, **29** 76 1921

[2] Some recent results of plate tests at the Harvard Astronomical Laboratory Pop Astronomy **30** 12 1922

the straight line portion can be used with advantage. The characteristics of method (*b*), curve B, on the other hand, are seen to be entirely different, the sensitivity at threshold being as great as any other portion of the curve, with a tendency to increase at the higher intensities. The total useful range of intensity in the most favorable case is found to be 5 or 6 stellar magnitudes (1 to 100 in intensity) for method (*a*) and about 10 magnitudes (1 to 10,000) for method (*b*). These results are based on laboratory measurements. Astronomers find the ranges considerably less than these, being respectively 3 to 4, and 5 to 6 magnitudes. It is not apparent why there should be this difference, especially in the case of method (*a*). While several explanations suggest themselves, it is unprofitable to speculate on the cause in the absence of data.

The increase in density of a photographic image and the increase in size, which respectively lie at the basis of the methods of photometry just outlined, are phenomena partly of common origin. The increase in size of an image, or spreading, as it is commonly called, in the case of a perfectly sharp image, is due to multiple reflection, refraction, and diffraction of the image-forming light by the grains of silver halide which in a thickly packed layer constitute the emulsion. The light is thus deviated from its original direction perpendicular to the plate, in a succession of steps until its direction of flow, measured by normals to the equiluminous surfaces, lies in the plane of the emulsion perpendicular to the edge of the geometrical

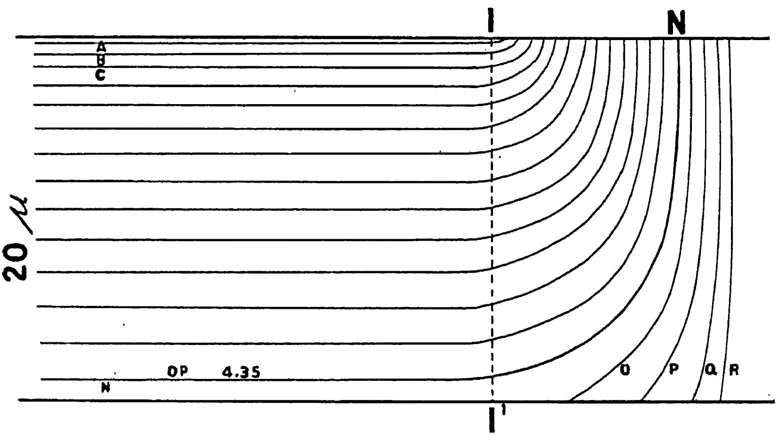


FIG. 11
Equiluminous surfaces in an emulsion.

image The equiluminous surfaces A - - - - N which lie under the image itself are parallel to the surface of the emulsion and can obviously be joined or linked up with the equiluminous surfaces which lie off the edge The photometric characteristics of the density of an image upon which method (*a*) is based are determined in part at least by the surfaces A - - N, while method (*b*) is based upon the characteristics of the surfaces A - - - - R - - - It is important to study these surfaces in more detail

Normal penetration of light in an emulsion There s a variety of factors which govern the penetration of light in an emulsion Among these may be mentioned size, shape, packing, orientation, and optical properties of the silver halide grain, properties of the embedding gelatin, wave-length and composition of the light, and thickness of the emulsion layer Concerning the specific action of each of these factors, little or nothing is known, although the subject is of great importance from many points of view

The silver halide grains composing an emulsion are transparent crystals embedded in a medium of lower refractive index The refractive indices in air and in gelatin are as follows

TABLE 3

	μD	μ (Gelatin)
Silver bromide	2 2533	1 458
Silver iodide	2 1816	1 410
Silver chloride	2 061	1 332
Gelatin	1 547	

From these numbers the following percentages of reflecting power in gelatin at normal incidence and the angles of total reflection are found

TABLE 4

	Reflecting Power	Angle of Total Reflection
Silver bromide	3 5%	43° 20
Silver iodide	2 9	45 17
Silver chloride	2 0	48 65

Thus, the silver chloride possesses considerable advantage over silver bromide in greater freedom from side scatter, resulting in a higher direct transmission If there were no reflection (or diffraction) in the passage of the light through the emulsion, the transmission would follow Bouguer's law, or Beer's law, as it is sometimes called, which states that equal thicknesses absorb equal percentages of incident light The

subject has been investigated theoretically and experimentally by Bloch and Renwick[1] who found considerable deviation from Bouguer's law. Calling D the effective density, where

$D = \log_{10}$ (Opacity); Opacity = reciprocal of Transmission, and putting W = weight of the silver bromide in milligrams per square centimeter, they find for a typical fast emulsion:

With white light illumination, $D = 0.66 \quad W^{0.64}$

With violet light illumination, $D = 1.123 \quad W^{0.82}$

If Bouguer's law held, the exponent of W would be unity. It is seen that a strong color effect is shown, as was to be expected. In addition to color differences variations in opacity were found depending on the particular conditions of measurement, which, however, it is not necessary to enter into here.

For the average fast emulsion $W = 1$ very nearly. The corresponding opacities from the formula are:

$$O = 4.6 \text{ blue illumination,}$$

$$O = 13.3 \text{ violet illumination.}$$

The equiluminous surfaces A - - - N shown in Fig. 11 are drawn accurately to scale, being based on the first of Bloch and Renwick's equation

$$D = 0.66 \; W^{0.64},$$

which holds for ordinary conditions of plate illumination. The illumination at each equiluminous surface is taken at 90 per cent of the value at the preceding surface, thus each layer absorbs 10 per cent of the light incident upon it. It is seen that the effective densities decrease downward, requiring a progressively greater thickness to produce the 10 per cent drop in illumination. Assuming a total thickness of the emulsion of 20μ, it can be calculated from the figure that 38 per cent of the incident light is absorbed in a layer 2μ thick adjoining the surface, while only 3 per cent is absorbed in a similar layer at the bottom. Assuming Bouguer's law, these values become 18 and 5 per cent, respectively.

The results obtained by Bloch and Renwick are in part confirmed by an earlier investigation by Abney[2] who found that an ordinary Kodak film transmitted 22 per cent of the incident photographic light (opacity = 4.5), agreeing remarkably well with the results computed from Bloch and Renwick's data, assuming W equal to unity.

[1] Bloch, O., and Renwick, F. F., The opacity of diffusing media. Phot. J. **40**: 49. 1916.

[2] Abney, W. de W., Effect of thickness of the film on the image and on the sensitiveness of the plate. J. Camera Club, **13**: 173. 1901.

Values of the transmission of various types of plates have been determined photographically in this laboratory by E Huse for the three principal colors, and white, with results shown in Table 5 The recording medium in all cases was a Wratten and Wainwright Panchromatic plate

TABLE 5

Plate	Transmissions in % White	Blue	Green	Red
Seed 30	22	8	39	41
Seed 23	18	8	32	39
Seed Process	34	11	40	36
Seed Lantern	41	25	43	44
Standard Orthonon	13		26	30
W & W Panchromatic	10	5	12	23

The blue, green and red in this table refer to the transmission of Wratten and Wainwright filters C, B, and A, respectively White refers to tungsten quality

Data on the optical behavior of emulsion film are given by Nutting,[1] who says (l c , p 392) "Undeveloped photographic plates have a high turbidity and for blue light a high opacity The emulsion with which the plate is coated is yellowish-white in color and roughly 0 02 mm thick It has a moderate surface gloss Distribution curves plotted for Seed 23, 30, and lantern plates with white light gave substantially the same results The brightness given is relative to that of a perfectly diffusing surface reflecting 100 per cent" His results are summarized in Table 6

TABLE 6

	Reflection					Transmission				
Angle	15°	30°	45°	60°	75°	105°	120°	135°	150°	165°
Brightness	0 69	0 56	0 54	0 52	0 43	0 27	0 33	0 35	0 37	0 37

These values are plotted in Fig 12, which shows approximately 57 per cent of the incident light is reflected and 35 per cent transmitted This leaves only 8 per cent as the amount absorbed, which seems rather low even taking into consideration the fact that the measurements are visual

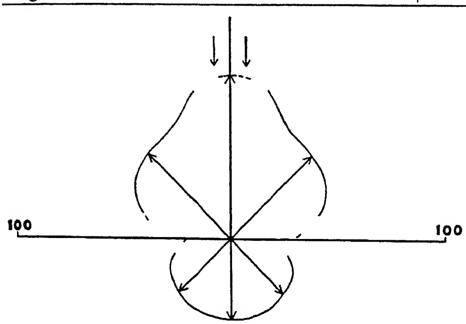


FIG 12
Distribution curve of reflected and transmitted light

[1] Nutting P G The optical properties of diffusing media Trans Ill Eng Soc 10 353 1915

Consideration of the equiluminous surfaces in a film will now be resumed Renwick's data give the distribution of the equiluminous surfaces directly under the image It remains to investigate the behavior of these same surfaces upon entering the shadow adjacent to the image where they must rise to the surface Their course will evidently depend upon the degree of sharpness of the image under consideration Only the ideal case of a perfectly sharp image, such as is produced by contact printing (see p 119) will be considered at this point In this case Fig 43 gives the light intensity at any given distance from the geometrical edge of the image for a number of wave-lengths Assuming, for example, a wave-length of 480μμ, it is found from the diagram that at a distance of 10μ from the edge the light intensity is reduced to 23 per cent of its value over the image Now this is very approximately the transmission at the surface N Accordingly, the equiluminous surface n, which is at a depth of 20μ, must rise to the surface at a distance 10μ from the edge I Knowing this one point of exit, all the other points of exit are at once obtained, for the intensity of the illumination at any point outside a sharp image is shown (p 94) to obey Bouguer's law It is only necessary then to divide IN into 14 equal parts and continue the subdivisions beyond n in the same proportion The points thus obtained are the exit points of the entire series of equiluminous surfaces A - - - R - The intermediate portions of the curves themselves can be only roughly sketched To obtain their true form would require an intricate mathematical investigation of the whole subject, involving many unknown parameters

The formula proposed by Tugman (p 120) for the light intensity in the region off the edge of the image, namely,

$$I = I_0 e^{-K(x+y)},$$

where x is the distance from the edge and y the depth or distance below the surface, is seen not to hold A much more complicated formula is necessary The rate of diminution of intensity downward appears to follow a different law than the law of sidewise drop in intensity From the figure, it is seen that in the ideal case chosen the equiluminous surfaces off the edge of the image are much more closely packed together than those lying under the image This can be explained as a color absorption effect The light which is traveling parallel to the surface of the emulsion must, on account of being composed largely of diffracted light, be of shorter average wave-length than the light passing normally through the emulsion The opacity in this direction will accordingly be higher, resulting

in a compression of the series of equiluminous surfaces In the case of images formed by optical systems the degree of compression, or distance between consecutive equiluminous surfaces, in the case of the vertical system of surfaces lying off the edge, will vary widely with the sharpness or perfection of the image The horizontal system (under the image) will be independent of all conditions except composition of the imaging light, and its converging angle For parallel light the opacity should be higher and the penetration less than for light impinging in a wide cone, corresponding to the condition of diffuse illumination

In Fig 11 it will be observed that the equiluminous surfaces rise to the surface in a nearly perpendicular direction, even for those near I where the intensity is high While no mathematical reasoning has been advanced to show that they are of the particular forms shown in the drawing, it is at least easy to see that the surfaces must begin to rise before the geometrical image edge II' has been crossed This is owing to the fact that there is a deficiency of illumination just off the edge, which affects the conditions of illumination just within the edge, causing a diminution of illumination there, and a consequent rise of the surfaces along this plane The point raised, however, appears to be definitely decided by experimental evidence Fig 13 shows a series of sections of sharp images, formed by contact printing of the ideal kind being considered The sections are swelled with alcohol and water, the vertical scale being about ten times the horizontal The weight of evidence from the nine sections is that the equiluminous surfaces are vertical, the image growing or spreading sidewise at the same rate at the bottom of the emulsion film as at the top This is for the ideal case of extremely sharp images Fig 14 shows a case in which this is not true This is a section of an image obtained in the same way, but with imperfect contact between the knife edge and the emulsion The equiluminous surfaces are seen in this case to be more nearly horizontal than vertical Intermediate types of all degrees of inclination can similarly be obtained In any case it can be specified that the form and orientation of the surfaces of equal illumination are the resultant of the surface distribution due to the optical system and the emulsion distribution caused by the spreading of the image-forming light The problem is accordingly an exceedingly complicated one, becoming more complicated as other factors, development factors, for example, are considered

We are now in a position to understand some of the principles underlying the two kinds of astronomical photometry

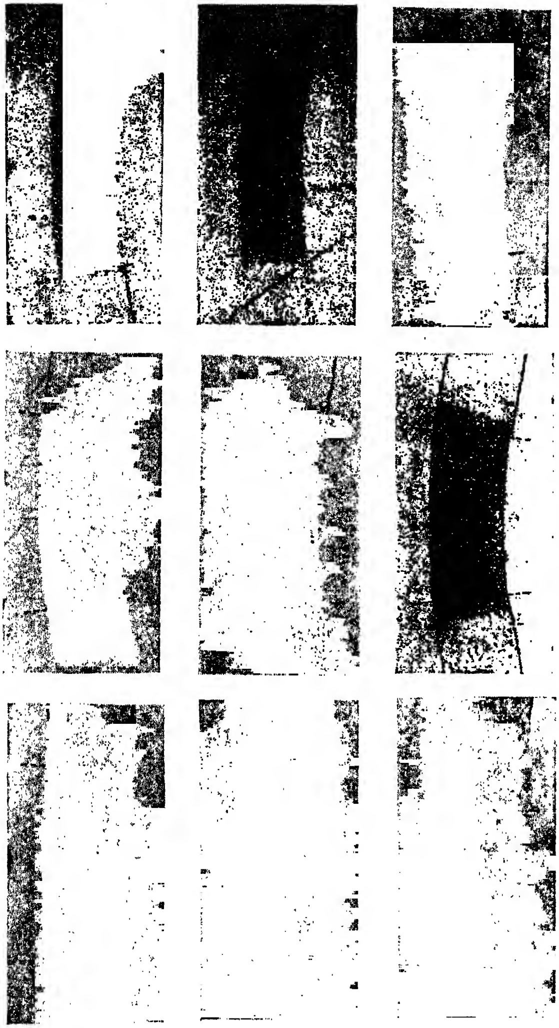


FIG. 13—Sections of sharp images

FIG. 14—Section of diffuse image

as well as their essential differences. In the densitometric method, photometric measurement is limited by the amount of silver which can be deposited in a vertical direction, which in turn is limited by the emulsion thickness. Obviously, when a long range of moderate sensitivity is desired, it is advantageous to have the specific opacity of the emulsion as high as possible. If, on the other hand, high sensitivity in photometric measurement is desired, regardless of a short working scale, an emulsion of low opacity is preferable. In the second method of astronomical photometry, in which the increase in size of a small image is made use of, obviously a much wider range of intensities can be measured, since there is no limit to the size which the image may attain (p. 89). The sensitivity of this method is, however, less than that of the first, as will be shown (p. 106). This might be inferred directly from the equiluminous surfaces of Fig. 11, in which the penetrating of the light downward is seen to be greater than that taking place sidewise, judged by the spacing of the equiluminous surfaces. Another factor of as great or greater importance in governing the relative sensitivity of the two methods is the greater accuracy in measuring the densities of photographic deposits within certain limits of density compared with measuring the size of an image which is surrounded by necessarily ragged borders. It is important to note, however, that the sensitivity of both methods is subject to control in a variety of ways. The sensitivity of the densitometric method depends upon gamma (p. 47) which depends upon the emulsion, upon developer and development time, and upon the wave-length of the light used. The sensitivity of the size-of-image method depends upon the emulsion, the wave-length of light, but not upon developer or development time (p. 105). An additional important factor in determining the sensitivity of the second method is the optical sharpness of the image, which in the case of astronomical observations depends to a great extent upon the steadiness of the air. A perfectly sharp image and perfectly steady air is a detriment to accurate photometric work of this sort, on account of the sharp images, which increase in size at such a slow rate with intensity increase, as to produce a correspondingly low sensitivity. Accordingly, on account of its increased aberrations, spherical and chromatic, a telescope of long focus is preferable to one of short focus for work of this sort. Of course the increased accuracy obtainable in measuring sharp images partly counterbalances this inherent deficiency.

A third method of astronomical photometry has been devised and successfully used by H. T. Stetson[1], which in principle is a combination of the two standard methods just outlined. The star image to be photometered is placed over the center of a circular hole in a metal plate, the hole being somewhat larger than the star disk. It is illuminated by means of a condensing system, and by a projection lens an enlarged image is thrown on the surface of a delicate thermopile. Relative amounts of energy absorbed by the star disks are thus obtained, which generally speaking, amounts to a determination of the *total* mass of developed grain in the image. Stetson finds that the stellar magnitude is proportional to the fourth root of the galvanometer deflection, so that the calculations are equally as simple as in the older methods. In a later paper[2] improvements in the apparatus are noted, as a result of which the probable error of a set of three readings on a star image is reduced from .025 magnitude to the extremely small value of .009 magnitude (0.8 per cent). Aside from the advantage of the method being physical and therefore free from personality, it will be noted that the method takes advantage of the total effect of the light action, or the three-dimensional effect, to which neither of the older methods can lay claim.

A fourth method has been employed extensively in recent years, especially in the determination of the magnitude of faint stars, too faint for out-of-focus determinations and which likewise lack the sharpness necessary for measurement of diameter. In principle the method is roughly equivalent to the Stetson method, the eye in this case replacing the thermopile. The following quotation from Chapman and Melotte[3] explains the method: "The measures are made by simple comparison with a scale of numbered and graded comparison stars, the smallest of which is quite faint and gray. . . Several comparison scales are in use, in order to meet the various qualities of definition on different plates, that one being chosen which best matches the images on the particular plate to be measured. The scales are cut out of negatives containing fourteen exposures on a selected star of duration so calculated as to give a magnitude interval between each exposure of about 0.25 magnitude; the actual intervals are, however, determined anew for each plate measured. The star to be

[1] Stetson, H. T., On an apparatus and method for thermoelectric measurements in photographic photometry. Astrophys. J., **43:** 253, 325. 1916.

[2] Stetson, H. T., Some recent improvements in thermoelectric apparatus for photographic photometry. Pop. Astronomy, **26:** 8. 1918.

[3] Chapman and Melotte, Photographic magnitudes of 262 stars within 25° of the North Pole. Monthly Notices, Roy. Astronomical Soc. **74;** 41. 1913.

measured and the comparison scale are viewed simultaneously in the eyepiece of a comparator by an arrangement of reflecting prisms Estimates are made to one-tenth of the intervals on the comparison scale " The probable error of a single magnitude estimate in this method is about $\pm$ 08 magnitude

It will be necessary at this point to go back some distance and study the mathematical relations existing between the density or degree of blackening of a photographic image and the intensity of light and exposure time which produces it Even though the mechanism of the action of light upon the silver halide grain is unknown, much can be done in the way of a mathematical presentation and formulation of observed phenomena The degree of blackening upon exposure to light is measured directly as transmission T, from which opacity O and density D may be computed The relations are

$$O = \frac{1}{T},$$

$$D = \log_{10} O$$

There are several reasons making density a desirable measure of photographic effect It is found, for instance, that a series of equidistant densities, or densities increasing in arithmetical progression, density being as defined above, produce visually a sensation or appearance of *equal steps,* in other words, visual response is, within certain limits a linear function of density (Fechners' Law) Again it is found experimentally, that within wide limits the relation between mass (M) of reduced silver per unit area (cm^2) and density is linear, of the form

$$M = cD, \tag{1a}$$

c being the photometric constant Its value is approximately 0 1 milligram for ordinary photographic negatives It is assumed that the viewing illumination in measuring density is completely diffuse

It can be assumed as a matter of experience that

$$D = f\,(I, t, \qquad), \tag{1}$$

or, density is a function of intensity, time of action, and other variables or parameters These parameters are so numerous that it is impracticable even to specify them For the present the reciprocity law will be assumed to hold, namely, that I and t occur only in product form

Before undertaking a mathematical discussion of the subject, a brief description will be given of the observed relations between D and $\log E$ With a given plate or emulsion, a

given developer and temperature, and a given quality of illumination, the relation between D and $\log E$ is shown in Fig. 15, which shows the curves for two similar and similarly

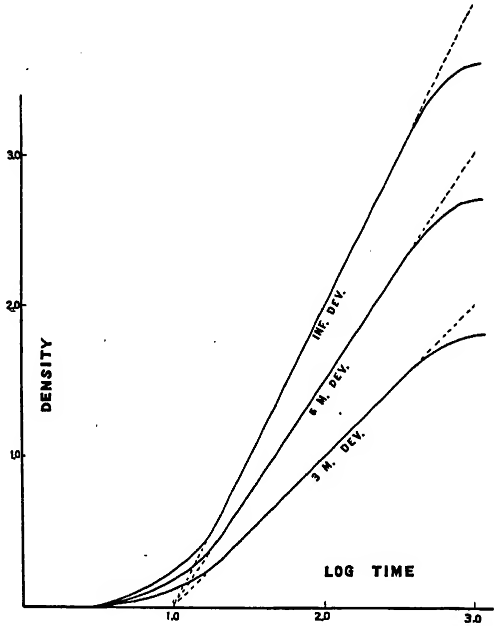


FIG. 15
H. and D. curves for two development times

exposed plates developed three and six minutes, respectively, in a pyro developer. It is seen that the relation is nearly linear over a considerable portion of the curves. The slope of the straight-line portion is designated as γ and is a measure of the contrast quality of the plate. It is found to increase with time of development in accordance with the relation:

$$\gamma = \gamma_\infty \left(1 - e^{-at}\right), \tag{2}$$

where a is a parameter measuring the velocity of development; γ_∞ is the maximum value of γ obtained from complete development. The curves in Fig. 15 are designated "characteristic" curves, or H. and D. curves, named after Hurter and Driffield, who developed the system of expressing the relations in this form.

Neglecting all formulae purporting to take account of the phenomenon of reversal, four specific formulae of the form (1)

have been proposed (*a*) by Abney[1] in 1889, (*b*) by Hurter and Driffield[2] in 1890, (*c*) by Elder[3] in 1893, (*d*) by Channon[4] in 1906 Abney's formula is purely empirical The formulae of Hurter and Driffield, of Elder, and of Channon are based upon differential equations specifying the velocity of the light action, or the velocity of infection of the silver grains, and are to that extent to be considered as theoretical equations

Abney's Formula The formula of Abney is

$$T = e^{-\mu\left(\log \frac{It}{i}\right)^2}, \tag{3}$$

or, in terms of density,

$$D = \mu' \log^2 \frac{It}{i} \tag{3a}$$

An example is given in Fig 16 of the character of the agreement to be expected between Abney's formula and observed curves In the diagram, transparency is plotted against the logarithm of the exposure It The full curves are of a commercial rapid plate, developed for one and three minutes respectively The dotted curves are computed from Abney's formula, the constants μ and i being so chosen as to produce the closest possible agreement with the measured curves The values of the constants are

	μ	$\log i$	γ
1 minute development	1 59	0 179	0 91
3 minute development	2 81	0 176	1 24

The quantity γ in the last column refers to the measured curves, and will be explained presently It will be noticed that i is independent of, while μ varies with, development time, i can be taken as a measure of the speed of a plate, while μ is a measure of the contrast value Agreement of observed and computed curves is fairly good from transparency unity to $T = 0\ 30$ ($D = 0\ 52$) in the case of one minute development, and as far as $T = 0\ 15$ ($D = 0\ 82$) for three minutes' development The formula has no value for lower transparencies, corresponding to higher densities In fact, Abney's formula can be considered to give a very good representation of the

[1] Abney W de W Photography and the law of error J Camera Club **3** 93 1889, Rapidity of plates *ibid* **7** 126 1893

[2] Hurter F and Driffield V C Photochemical investigations and a new method of determination of the sensitiveness of photographic plates J Soc Chem Ind **9** 455 1890

[3] Elder, H M Some notes on the effect of light on photographic plates J Camera Club **7** 131 1893

[4] Channon H J A new formula for expressing density in terms of exposure Phot J **30** 216 1906

relation between transparency or density, and exposure, for the first step in the process, which covers the appearance of the image and its development in strength up to the point where rapid action begins. For certain character of work making use of weak or so-called under-exposures, Abney's formula applies more accurately than any other which has been proposed. There does not appear, however, to be any theoretical basis for it, at least none has been proposed. It fails to take account of the very obvious fact that there is a limit to the density which can be reached depending upon the amount of silver in the emulsion, the relation being given by equation (2) for γ.

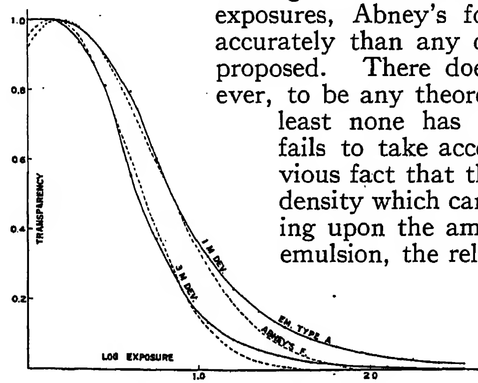


FIG. 16
Comparison of observed characteristic curves with Abney's formula

Elder's Formula. On account of its simplicity, Elder's formula will be considered next, although preceded in point of time by the more complicated Hurter and Driffield formula. Elder's equation is

$$D = D_m\left(1 - e^{-\kappa It}\right), \tag{4}$$

which is derived by integration of the differential equation,

$$dn = \kappa I(N - n)\,dt, \tag{4a}$$

where D_m is the maximum density obtainable, and is a function of the amount of silver in the emulsion; N is the total number of grains per cm.^{2}, to which D_m is proportional; n is the number of grains changed to latent image in the time $t = 0$ to $t = t$; I is light intensity, κ is a parameter depending principally upon grain-sensitivity and wave-length of light, and is a measure of the speed of the emulsion.

Equation $4a$ asserts that the number of grains changed to latent image in any small interval of time is proportional to the number of grains not yet acted upon. It is a well known formula for the velocity of chemical reaction. In making use of this equation, one is not necessarily committed to a chemical theory of the latent image, for the photoelectronic theory of the formation of the latent image leads to a similar equation[1]. For

[1] The photoelectronic theory has since been developed in great detail by Dr. L. Silberstein in a series of papers in the Philosophical Magazine. See p. 81

a very suggestive account of the photoelectronic theory the reader is referred to H S Allen's "Photoelectricity"

Comparison of formula 4 with observed curves is made in Fig 17 Photographic density D (diffuse) is the ordinate, logarithm of the exposure log E, is abscissa These H and D or characteristic curves[1] are not to be confused with the curves resulting from the Hurter and Driffield formula

It is seen that the parameter D_m in equation 4 controls the height of the theoretical curves (Elder) in Fig 17, while the parameter κ controls the position of the curve along the exposure axis, and is accordingly a measure of the speed of the emulsion D_m and κ were chosen in each case to agree with the maximum density and average position along the exposure axis of the observed curves, thus giving the closest possible agreement It will be seen that the agreement is not at all good, but is better in the case of the emulsion of Type B than in that of Type A

These emulsion types need a few words of explanation A characteristic curve consists of three parts (1) a portion of low density and slow change in density, called the "toe" of the curve, (2) a straight, or nearly straight portion comprised between the regions of low and high density, in which the *change* of density reaches a maximum, called the straight-line portion The maximum slope is called gamma (γ) and is an important constant in an emulsion, as it determines the contrast or power to reproduce shades in lighting It is the factor controlling sensitivity in photographic photometry Mathematically, gamma is given by

$$\gamma = \left(\frac{dD}{d \log E} \right) \text{Max} , \tag{5}$$

(3) the third portion of the characteristic curve is called the shoulder, and lies above and beyond the straight-line portion It is a region of high density and slow change of density Briefly, the toe is the region of underexposure, the straight-line portion is the region of correct exposure, and the shoulder is the region of overexposure

An emulsion of Type A is one having a very short "toe" Type B is one having an extended toe Similar differences are to be found in the shoulder, but generally they are not of such importance These differences in the toe in emulsions of various types are of importance in the subjects of photographic resolving power and sharpness

[1] The curves are made from measures of density on a plate which has been given a graded series of exposures in an instrument called a sensitometer For a description see paper by L A Jones A new non-intermittent sensitometer J Frankl Inst **184** 303 1920

If equation 4*a* is a complete description of the formation of the latent image, assuming regular and proportional development, computed and observed curves in Fig. 17 ought to coincide. As already remarked, this is found not to be the case. Only two parameters are at our disposal in equation 4. A third parameter appears to be necessary. At the time the equation was proposed, the question was raised whether it should not be modified to take account of the thickness of the emulsion. Elder[1] advanced what purported to be a proof that the equation as it stands virtually takes account of film thick-

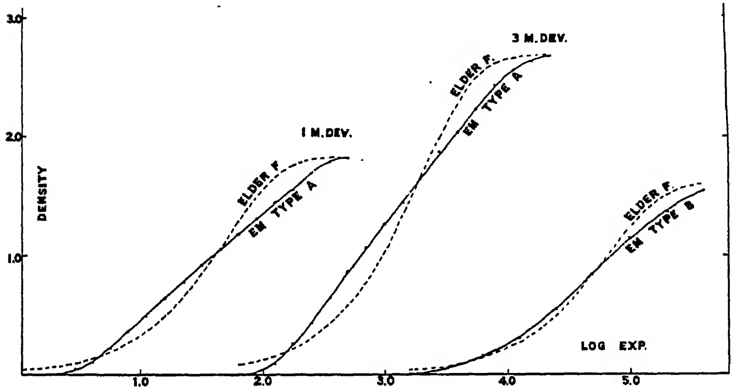


FIG. 17

Comparison of observed characteristic curves with Elder's formula

ness, it being only necessary to multiply the light intensity in equation 4a by the factor $\frac{a^{s}-1}{a \log e}$, where a is the absorption coefficient of the film and s the thickness. Accordingly, the form of equation 4 should be unaltered if his demonstration is correct. The fallacy in the proof lies in the statement, " . . . the action, other things being equal, is proportional to the intensity of the light acting . . . ." It will be shown in a later paragraph that the thickness of the emulsion profoundly alters the characteristic curve and introduces a third parameter necessary to fit observation with theory. It can be concluded that the Elder equation fails in general to explain the characteristics of emulsions.

Hurter and Driffield Formula. The Hurter and Driffield equation will now be considered. The differential equation

[1] Elder, H. M., A reply to Abney's article on "The speed of plates and the effect of light on plates." J. Camera Club 7: 165. 1893.

upon which it is based is

$$dx = \frac{I}{b}\,(1-r)\left(e^{-\kappa x} - e^{-\kappa}\right)dt, \tag{6}$$

where dx is the number of grains per cm ^{2} changed to latent image in the time-interval dt, b the energy necessary to change one grain, I the light intensity, r the reflection coefficient, α the total number of grains per cm ^{2} in the emulsion, κ is a parameter to be considered later The integral equation of 6 is easily found Combining parameters and constants and making use of equation $M = cD$ (p 46), there results

$$D = p\log_{10}\left(O-(O-1)e^{-qIt}\right), \tag{7}$$

where $O = e^{-\kappa\alpha}$ There are three parameters in 7, one more than in equation 4, so that it can be expected to give closer agreement with observed curves than is possible in the case of the Elder equation

Equation 7 will now be applied to the two emulsion types A and B used in previous comparisons In fitting equation 7 to plotted curves, the parameters O and p are determined by the following formulae

$$\left.\begin{aligned} p\log_{10} O &= D_m, \\ p &= \frac{\gamma}{1-e^{-\log O}} \end{aligned}\right\} \tag{8}$$

The parameter q is determined by making the mid-point of the straight-line portion a common point of the observed and computed curves The constants and parameters found in this way are as follows

	Em A	Em B
D_m	2 70	1 60
γ	1 34	1 00
O	40 46	10 72
p	1 675	1 555
q	0 00127	0 00235

The curves (H D F) in Fig 18 were computed from the above numbers and plotted in conjunction with the observed curves In the case of Emulsion A, disagreement in the toe and shoulder is very marked With Emulsion B agreement is much better Comparing with Fig 17 it is seen that the Hurter and Driffield formula conforms to actual emulsions considerably better than the Elder formula

It will be worth while to examine the assumptions underlying the Hurter and Driffield formula The differential equation (6) is based on energy relations The reasoning is as follows If I is the intensity of the light incident on the plate, corrected for reflection loss, and T the transparency of the emulsion, the amount of energy absorbed by the plate in unit time is obviously

$$E_a = I\,(1 - T_a), \tag{9}$$

it being assumed that there are a grains of silver halide per cm² in the emulsion The subscripts in (9) and following formulae are simply descriptive of the particular mass of grains considered

Let the number of grains changed to latent image after a time t be denoted by x Denote the transparency of this group of grains by T_x The energy absorbed by this group in unit time is said to be

$$E_x = I\,(1 - T_x) \tag{10}$$

Let the amount of energy necessary to change one grain of silver bromide to latent image be denoted by b It is claimed that the number of grains dx so changed during the time dt will be given by the relation

$b\,dx$ = amount of energy absorbed by the *unchanged* silver grains, (11)

from which

$$b\,dx = (E_a - E_x)\,dt \tag{11a}$$

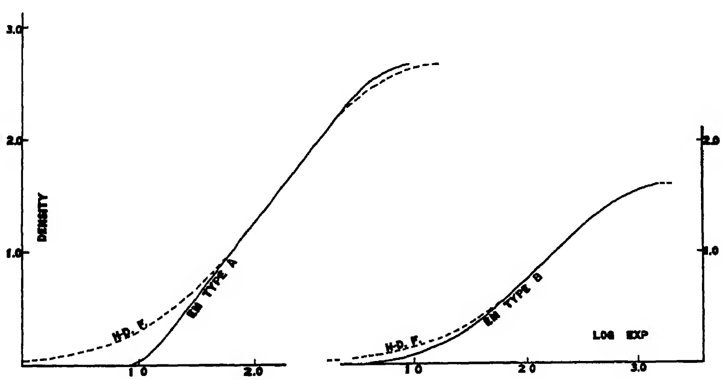


FIG 18
Comparison of observed characteristic curves with Hurter and Driffield's formula

The truth of equation (11a) can not be admitted It is not clear how the energy absorbed by the group of grains $a - x$ can be applied to the sub-group dx

Continuing, the next step is the substitution of the following values in (11a)

$$T_a = e^{-Ka}, \tag{12}$$

$$T_x = e^{-Kx} \tag{12a}$$

In writing equation (12a) serious error is made Equation (12) is undoubtedly valid, at least as an approximate equation If (12a) is true, it is equally true with regard to the group of grains $a - x$, i e,

$$T_{a-x} = e^{-K(a-x)}, \tag{13}$$

and equally with (10),

$$\mathrm{E}_{a-x} = I\,(1 - T_{a-x}) \tag{14}$$

We have, however,

$$E_a = E_x + E_{a-x}, \tag{15}$$

or,

$$I\,(1 - T_a) = I\,(1 - T_x) + I\,(1 - T_{a-x}),$$

substituting the assumed values of T there results

$$e^{-Ka} - e^{-Kx} = e^{-Ka}\,e^{-Kx} - 1,$$

which is true only for $x = 0$ The assumption of Hurter and Driffield that the transparency of the x-group of grains is given by e^{-Kx} is accordingly incorrect, for it leads to a mathematical inconsistency Their equations (6) and (7), in consequence, must be considered purely empirical

Channon's Formula Channon follows Elder in using the mass-action equation, but takes into account emulsion thickness The equation to which he is led is not directly integrable By expansion in series his solution takes the form

$$D = \gamma \left\{ \phi\,(aE) - \phi\,(aE\theta) \right\} \tag{16}$$

where γ, a, and θ are parameters, E is exposure, and ϕ a functional form as follows

$$\phi(n) = n - \frac{n^2}{2\lfloor 2} + \frac{n^3}{3\lfloor 3} - \frac{n^4}{4\lfloor 4} +$$

Agreement with observed curves is of the same order as the H and D formula Theoretically, it is well grounded, but fails

to take account of varying sensitivity of grain, which, in the case of fast emulsions, appears to have a greater effect upon the characteristic curve than the opacity factor

Development of a New Formula On account of the general applicability of the mass-action equation to certain classes of chemical and physical problems, it seems worth while to apply it to the present problem in a manner more circumspect and cautious than has hitherto been done It will be assumed that the law applies only to the case of a single layer of grains, which therefore individually receive equal exposures Actual emulsions are composed of grains lying in layers many layers thick and very often of grains of varying sensitiveness Obviously, many formulae can be constructed, depending upon particular assumptions of the following classes

a Relative frequency of grains of varying size,

b Relative sensitivity of grains of varying size,

c Variation of grain-sensitivity with light-intensity and wave-length for each class of grains

d Ratio of inert to active grains, depending on grain size

e Number of layers of silver halide

f Opacity of the emulsion and its variation with wave-length

This is not an exhaustive enumeration of the factors which must be considered in developing the theory of the subject. It would be unprofitable in the absence of data on the controlling factors enumerated above to develop a formula on any but the simplest assumptions These are

1 All the grains in an emulsion are divisible into n groups, each group as a whole obeying the mass-action law

2 The silver mass is the same for each group

3 The sensitivity factor (k) of the groups is arrangeable in geometrical progression This takes account of the factors of (a) true sensitivity differences, (b) apparent sensitivity differences, due to the thickness of the emulsion This equivalence is possible on account of the reciprocal equivalence of k and I in formula (4a)

With these assumptions, the equation connecting density with exposure can easily be derived Calling d_m the maximum density (for infinite exposure), s the sensitivity factor, r the common ratio of the sensitivities of the various groups, the equation becomes

$$D = d_m\left(1 - \frac{1}{n} \sum_{s=0}^{s=n-1} e^{-\kappa r^S It} \right) \tag{17}$$

In order to test formula (17), it will be necessary to give values to the parameters r and n If the values $r = \frac{1}{2}$ and $n = 1, 2, 3 \qquad 10$ are assigned, the field of practical emulsions should be well covered For, neglecting real sensitivity differences, the opacities, including real sensitivity differences, of our ideal emulsions would in this case vary from unity to $2^9 = 512$, a variation more than sufficient to include all emulsions

Fig 19 shows a series of curves calculated from equation (17), for $r = \frac{1}{2}$, and $d_m = 5$, for successive values of $n = 1$ to $n = 10$ The curves for any other value of d_m can be obtained from these curves by multiplying the ordinates by $1/5\ d_m$ It will be noticed that for increasing values of n the contrast or gamma becomes less and the straight-line portion longer

The maximum value of gamma is of especial interest Its value, easily obtained by differentiation, is as follows

$$\gamma_{(\max)} = \frac{1}{e\ Mod}\ D_{(\max)} = 0\ 847\ D_{(\max)} \tag{18}$$

If this numerical relation between gamma and maximum density should be found to hold for thin emulsions approximating one grain layer in thickness, the emulsion itself being of simple type, or for X-ray exposures on a similar emulsion of any degree of thickness, it would be valuable confirmation of the applicability of the law of mass action, which, in its essence, limits the exposure to a probability function related to the mechanism of light action, for example, a quantum theory, to inherent sensitivity variations of sensitivity nuclei in the grains themselves, or to a combination of the two

It is to be noted that the toe of the curve, a region of especial importance and interest, in Fig 19, flattens out with increase of n, a characteristic not indicated in the Hurter and Driffield theoretical curves It must be admitted, however, that in actual emulsions a wider range in the shape of the toe or underexposure region is encountered than any formula so far proposed would indicate A variety of reasons for this suggest themselves which it would be profitless to discuss without more data

In photographic literature the latitude of an emulsion is defined as the antilogarithm of the projection of the straight-line portion of the characteristic curve on the log-exposure axis It will be convenient to define a term range, of similar properties to latitude, in order to meet the objection that a mathematical curve has no true latitude By range is meant the

antilogarithm of the projection of the line AA' or BB' on the exposure-axis. It can be taken as a measure of the reproduc-

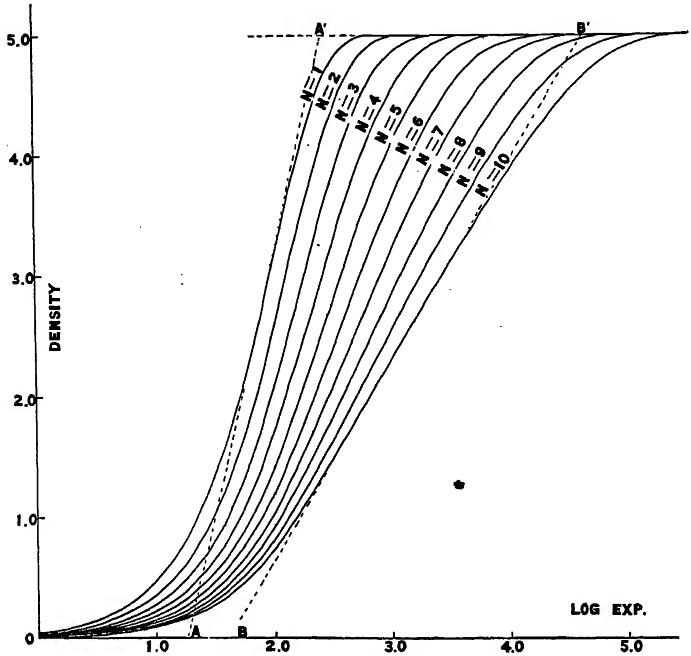


FIG. 19
Characteristic curves from new formula

ing power of an emulsion and is accordingly a useful and important function. By measurements made on Fig. 19 it is found that the range has the following values:

n	*Range*	n	*Range*
1	15	6	90
2	17	7	160
3	25	8	290
4	35	9	530
5	55	10	1000

Fig. 20 shows the first derivative curves for the same series of emulsions, obtained by differentiating equation (17) with respect to the logarithm of the exposure. The maximum ordinate of each curve is the gamma of the corresponding emulsion. From the broadening out of the curves with

increasing n it is seen how the range increases, and gamma decreases, with emulsion thickness The relation between gamma and n can be empirically derived from this series of curves However, gamma derived from Fig 20 will be found slightly greater than if obtained from the straight-line portion

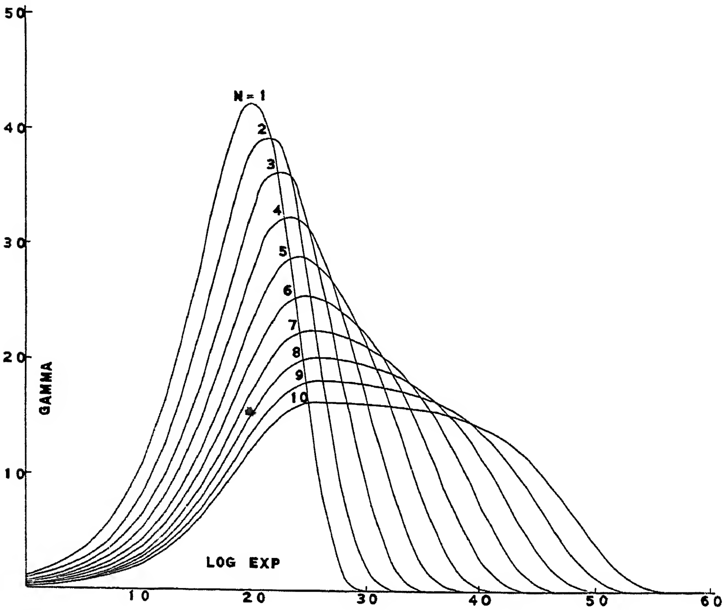


Fig 20
Slope curves from new formula

of the curves in Fig 19 In comparing theory with observed curves, it will be better to use values obtained from Fig 19 They have accordingly been scaled off as accurately as possible, plotted, and a smooth curve drawn The result, an exponential curve, is shown in Fig 21 reduced to $d_m = 1$

It remains to apply formula (17) to the emulsion types A and B, which have been used in the previous comparisons This can be easily done with the help of Fig 21 From the characteristic curves of A and B, γ and d_m, are known With the ratio as ordinate, n is obtained directly from the figure We have

	γ	d_m	γ/d_m	n
Emulsion A	1 34	2 70	0 496	6
Emulsion B	1 00	1 60	0 625	4

Scale off the ordinates of the curves in Fig. 19 for $n = 6$ and for $n = 4$. Multiply each value by $d_m/5$. The result is the theoretical characteristic curves, which are given in Fig. 22. Agreement with the observed curves is seen to be very good in the case of Type B.

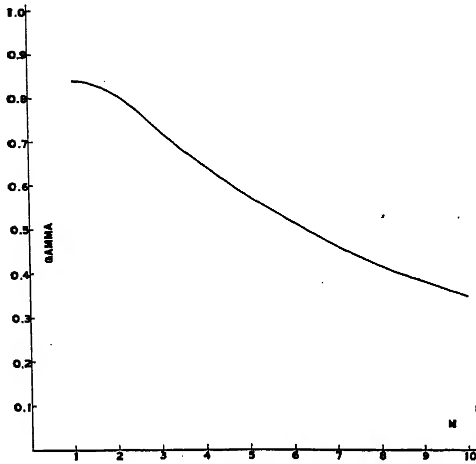


Fig. 21
Variation of Gamma with n

It is quite evident that there is an appreciable secondary action depending on the type of emulsion, which depresses the initial portion of the curve to a greater or less extent, depending upon the emulsion. Translated in terms of light action, this means that there is an inertia period — possibly a chemical change of some sort produced by the light is necessary before the normal action is effective. The other alternative is that it is a developer effect. There is evidence that the latent image forms at numerous points of the silver halide grain, which increase in number and size with exposure. To explain

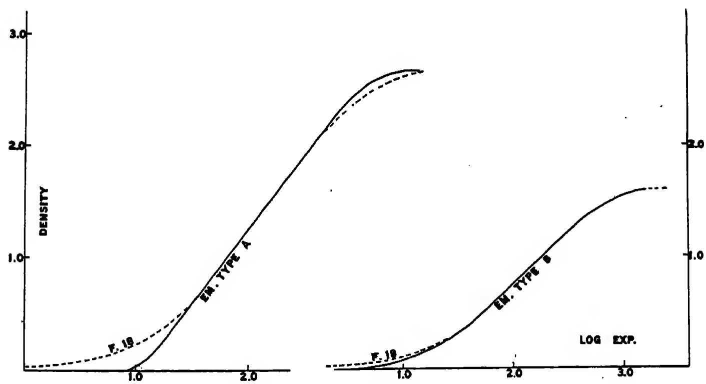


Fig. 22
Comparison of observed characteristic curves with new formula

the observed depression of the curve it is only necessary to assume that a grain does not fully develop until the latent image connected with it has reached a certain phase The effect must vary with the emulsion It is necessary to state here that the observed characteristic curves shown in this paper have all been obtained from a time-scale, or t variable Curves resulting from an intensity scale are not at hand They should coincide with the time-scale curves if the reciprocity law holds If Schwarzschild's law holds, they will differ in the density-ordinate only by a common factor p, and must be accordingly of the same form, as will be shown It is necessary to collect further data on the subject, however, before definite conclusions can be reached

ADDENDUM

The full characteristic curve of plates The H and D or characteristic curves which are described in the preceding are, as will be noted, limited to a maximum density of about 3 0 In the measurement of densities it has not hitherto been possible to obtain density values greater than this, on account largely of scattered light in the optical system designed for making the measurements A high-intensity densitometer has recently been designed and constructed for this Laboratory by L A Jones[1], by means of which densities as high as 6 0 (diffuse) can be accurately measured It will be of interest and importance to give the full or complete characteristic curve obtained with this instrument for a Seed 23 plate Exposure was made to an intensity scale, and development in MQ process developer for 3 minutes Longer development would have resulted in higher maximum density and gamma The mean results are

log E	Density
0 6	0 13
1 0	0 57
2 0	1 13
3 0	2 21
4 0	3 30
5 0	4 15
6 0	4 20
7 0	3 90
8 0	3 00
8 3	2 55

[1] Jones L A An instrument (Densitometer) for the measurement of high photographic densities J Opt Soc Amer 7 231 1923

The illumination for the highest exposure was approximately 20,000 meter-candles, the constant exposure time, 60 seconds. The range of exposure is seen to be 50 million (antilog 7.7). The straight line portion has a remarkable length, with a latitude of 10,000 (antilog 4.0). Reversal is seen to commence at the point of maximum density—there is no region of constant maximum density, which, when obtained heretofore, was quite likely due to inability to measure the upper portion of the curve, as explained above. It is to be pointed out that these curves can not be used as they stand in theoretical investigations, of the nature of those occupying our attention in the preceding pages, until the relation between mass of silver and diffuse density (page 46), is obtained for densities over 3.0.

THE RECIPROCITY LAW

The early experimenters found that the same photographic effect could be obtained by reciprocal variations of light-intensity and exposure-time. The relation, therefore, assumed the comparatively simple form

$$D = f\,(E)\,;\; E = It.$$

It will be useful to assume that density D can be expressed as a function f of a single variable U and then find from experimental data the relation between U and the variables I and t. With this assumption

$$D=f\,(U),$$
$$U=\phi(I,t).$$

Differentiating,

$$\delta D = \frac{dF}{dU}\left(\frac{\delta\phi}{\delta I}\,\delta I+\frac{\delta\phi}{\delta t}\,\delta t\right);$$

or, expressed in logarithmic increments,

$$\delta D = \frac{dF}{dU}\left(I\,\frac{\delta\phi}{\delta I}\,\delta(\log I)+t\frac{\delta\phi}{\delta t}\,\delta\,(\log t)\right). \qquad (20)$$

Assume that a pair of equal densities D_0 are produced by two pairs of values of the variables I and t; *i.e.*,

$$D_0 = F\,(U_0),$$
$$U_0 = \phi\,(I_1;\, t_1) = \phi\,(I_2;\, t_2).$$

What is the condition that ∂D in 20 shall depend only on U or on ϕ? It is clearly

$$\left.\begin{aligned} t\frac{\partial \phi}{\partial t} &= \varphi(\phi), \\ I\frac{\partial \phi}{\partial I} &= \chi(\phi) \end{aligned}\right\} \qquad (21)$$

If a power series be assumed to satisfy (21) it will be found to reduce to a single term

$$\phi = I^m t^n = I t^p, \qquad (22)$$

or, alternatively,

$$\phi = \log (I^u t^v) \qquad (22a)$$

It is important to state the physical meaning of equation (21) If this equation holds, the variables I and t do not occur explicitly in the coefficients in the right-hand member of equation (20) In the case assumed, of two equal densities D_0 produced by the same value of U but different values of I and t, the increments ∂D which they undergo from equal increments of log I or of log t accordingly must be identical In order that this proposition may extend to finite values of density change, it is further necessary that $t^n \frac{\partial^n \phi}{\partial t^n}$ and $I^n \frac{\partial^n \phi}{\partial I^n}$ shall be functions of ϕ. Formulae (22) and (22a) are seen to satisfy this criterion The interpretation is as follows

Suppose a series of densities D_0, D_1 D_m are produced by a series of exposures in which either the time or the intensity increases in geometrical progression (logarithmically), *i e*,

time or intensity	$\left\{\begin{matrix} t \\ I \end{matrix}\right.$	$\begin{matrix} 2t \\ 2I \end{matrix}$	$\left.\begin{matrix} 4t \\ 4I \end{matrix}\right\}$
density produced	D_0	D_1	D_2

Suppose another series produced in the same way, with this difference, that the initial density D_0' is produced by an entirely different combination of I and t than obtains in the first series We have then

time or intensity	$\left\{\begin{matrix} t' \\ I' \end{matrix}\right.$	$\begin{matrix} 2t' \\ 2I' \end{matrix}$	$\left.\begin{matrix} 4t' \\ 4I' \end{matrix}\right\}$
density produced	D_0'	D_1'	D_2'

The theorem just proved asserts that if $D_0 = D_0'$, we must have

$$\left.\begin{aligned} D_1 &= D_1' \\ D_2 &= D_2' \end{aligned}\right\} \tag{23}$$

Or, in words, a series of densities produced by logarithmic variations of I or t is independent of how the initial density in the series is produced. Expressed otherwise, if two series of densities are produced in this way, if any one member of one series coincides in density with any one member of the other series, then the two series will coincide throughout.

To what extent are equations (21) and (22) realized in practice? The earlier experimenters found the very simple expression

$$\phi = I\,t, \tag{24}$$

which satisfies (21). Schwarzschild and others have since found

$$\phi = I\,t^p, \tag{25}$$

which also satisfies (21). On the other hand, E. Kron[1] has found

$$\phi = t\,I\,10^{-a\sqrt{\left(\log \frac{I}{I_0}\right)^2 + 1}}, \tag{26}$$

in which a and I_0 are parameters depending upon the emulsion. Applying (21) to this equation gives

$$t\frac{\partial \phi}{\partial t} = \phi,$$

$$I\frac{\partial \phi}{\partial I} = \phi\left\{1 - \frac{a}{Mod}\log\frac{I}{I_0}\left[\left(\log\frac{I}{I_0}\right)^2 + 1\right]^{-\frac{1}{2}}\right\} \tag{26a}$$

Accordingly only the first of (21) is satisfied. This means that according to Kron's experimental results the time scale holds, but not the intensity scale. In other words, the two series of densities considered above

$$D_0 \quad D_1 \quad D_2,$$

and

$$D_0' \quad D_1' \quad D_2',$$

are equal for time variations, but not for intensity variations. This is a departure from results so far obtained and should be confirmed by numerous experiments.

[1] Kron, E., Ueber das Schwarzungsgesetz photographischer Platten. Jahrb. Phot. 28, 6, 1914, and Publ. Astrophys. Obs. zu Potsdam No. 67, 1913.

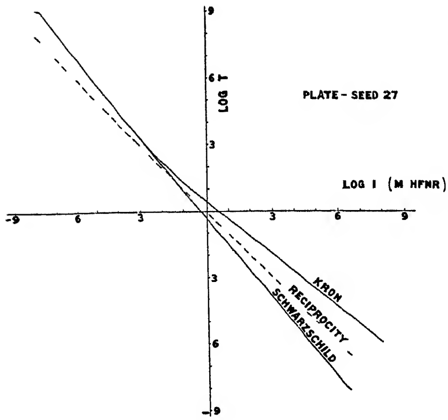


FIG 23
Curves of equal blackening

The relation between log I and log t to produce equal blackening in the respective cases of the reciprocity law, of Schwarzschild's, and of Kron's law holding is plotted in Fig 23 for Seed 27 plates, for which the parameters a and log I_0 are given by Kron They are

$a = 0\ 21$, $I_0 =$ 013 meter-Hefner

Kron shows, which can easily be verified, that his law becomes identical with Schwarzschild's for low intensities, the relation between the parameters being

$$p = \frac{1}{1+a}$$

From the figure this is seen to be true Study of the curves is instructive If Kron's results are correct, the reciprocity law holds only for a very limited range of intensities, one to ten at the maximum, in the region of optimal intensity, the values of which for four emulsions, are given in table 7 below Furthermore, for all higher intensities, the reciprocity law holds with more exactness than Schwarzschild's

If Schwarzschild's law is correct, the reciprocity law does not hold for any value of the intensity, the error being the same in percentage amount for all intensities A practical example is as follows Suppose in a given case that a certain blackening is produced, and that an equal blackening is desired when the intensity is decreased a hundred-fold, the exposure-time necessary will be 2 4 times the value computed from the reciprocity law, assuming $p = 0\ 83$ as in the figure

In Fig 23 log I is plotted against log t If on the other hand log It is plotted against log I, some interesting and useful relations are brought out In the case of Kron's law, the only one of interest in this connection, one of the series of parallel curves is shown in Fig 24 for Seed 27 plates, the curve being a hyperbola From this figure, the remarkable theorem is at

once apparent, that to produce any given photographic effect there is a certain intensity for which the energy necessary (It) is a *minimum*. This value of the intensity is called by Kron the optimal intensity (optimale intensität). The exposure

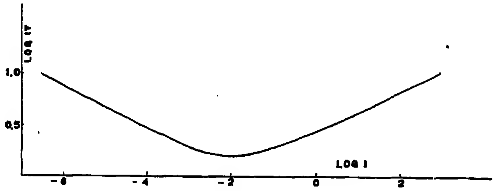


FIG. 24

time for optimal effect for an average density is found to be about two minutes. The optimal intensity is easily seen to be equal to I_0 in formula (26). The values of I_0 and a for various plates determined by Kron are:

TABLE 7

Plate	a	I_0 candles (Hefner)
Agfa diapositiv	0.22	1.7
Ideal diapositiv	0.11	0.59
Schleussner blue	0.19	0.042
Seed 27	0.21	0.013

According to Kron, therefore, there is an intensity most economical of energy in producing any given photographic effect. With the reciprocity law holding, the intensity value is indifferent. With Schwarzschild's law operative there is a continuous gain of efficiency for higher intensities, no limit being reached as is easily seen by slightly transforming the Schwarzschild equation. That there should be an optimal intensity is indeed a remarkable fact, the physical interpretation of which is at the present time beyond us. A partial explanation which at once suggests itself is that for high intensities the exposure time is so short that the intermittency correction becomes sensible. The shortest exposure time employed by Kron was five seconds, which is longer than usually associated with the intermittent effect. The loss of efficiency for values of the intensity less than I_0 is not so readily explained.

One of the most remarkable results obtained by Kron is a shift in the optimal intensity, depending on development time, in general an increase in time of development shifting the optimal intensity to a higher value. The curves given by

Kron show for the first three emulsions of Table 7 an increase of a hundred-fold in optimal intensity, for Seed 27 an increase of but ten times. This means that the failure of the reciprocity law can not be specified in terms of light intensity, but that developer and development time must be taken into consideration. As Kron points out, it follows as a corollary that if the values of I and t are adjusted to give equal densities at one development time, the densities will become unequal for another time of development. It may accordingly be inferred that the relative velocities of development must be different in the two cases, depending on whether the images are formed by strong or weak light respectively. This effect, which is analogous to a generalized Purkinje phenomenon, may appropriately be called the Kron effect.

Kron's law of photographic action has been applied with great success by Halm,[1] who finds abundant evidence of its correctness in a careful analysis of a mass of astronomical photometric data.

The factors upon which the exact laws of photographic photometry depend are probably the following: (*a*) character and homogeneity of the emulsion, (*b*) emulsion thickness, (*c*) wave-length of light, (*d*) development factors. If it is true that these factors are of importance, and theoretical considerations indicate that they are, it is not surprising that concordant results have not yet been obtained.

So long as the reciprocity law of photographic action was supposed to hold, the character of the fundamental process of latent image formation appeared to be simple. Since energy absorbed is the product of intensity and time of action ($E = It$) the formation of the latent image appeared to be only a function of the energy absorbed by the silver grains. In other words, exposure should be merely a matter of energy absorption. It was from this standpoint that Hurter and Driffield developed their equation (7) of photographic action.

There are two precision methods available for determining the exponent p in equation (22) as follows: (1) measurements of density, (2) measurement of diameters of star images. A discussion of the second method will be given later (p. 105). The first method is based upon a simple formula, which can be derived as follows. Assume

$$D = f(U), \quad U = It^p$$

[1]Halm, J. On the determination of photographic magnitudes. Monthly Notices Roy. Astronomical Soc. 75, 150. 1915, *ibid* 78, 379. 1918.

Differentiation gives

$$\frac{\delta D}{\delta \log I} = I\frac{df}{dU}\frac{\delta U}{\delta I} = U\frac{df}{dU}, \tag{27}$$

and

$$\frac{\delta D}{\delta \log t} = t\frac{df}{dU}\frac{\delta U}{\delta t} = pU\frac{df}{dU}; \tag{28}$$

from (27) and (28),

$$\frac{\delta D}{\delta \log t} = p\frac{\delta D}{\delta \log I}; \tag{29}$$

or since

$$\frac{\delta D}{\delta \log t} = \gamma_t, \quad \frac{\delta D}{\delta \log I} = \gamma_I, \tag{30}$$

we have

$$\gamma_t = p\gamma_I.$$

Accordingly p is the ratio of the gammas of the characteristic curves obtained by increasing the exposure by time-increments and by intensity-increments, respectively. Since it is possible to measure gamma with considerable precision, an accurate value of p should be obtainable.

A second method of determining p by sensitometric data is to plot the series of characteristic or H. and D. curves, the individual members of which are made on a time scale, the intensity varying from curve to curve in a continuous series of steps. Let I_1 and I_2 denote the intensities corresponding to any two curves, consecutive or non-consecutive. Let the corresponding displacement of the curve for I_2 relative to that for I_1 measured along the log-exposure axis be S. It is easy to prove that the Schwarzschild exponent is given by the equation

$$S = \frac{1-p}{p}\log\frac{I_1}{I_2},$$

which shows that S is independent of the time t, and accordingly the curves are strictly parallel. In addition it is seen from the equation that if p is less than unity, the curves move outward for diminishing intensity I. If on the other hand p is greater than unity, the curves move inward. If p is unity,

there is no movement, and the curves are coincident If Schwarzschild's equation is valid, p must be a constant, in which case the curves move uniformly outward for all variations of I With Kron's law holding it should be found experimentally that for decreasing intensity, starting with very high intensity, the curves first move inward, corresponding to p greater than unity, reaching a stationary point at the intensity for which p is unity, from which point the motion is outward as the intensity decreases, corresponding to p less than unity

A similar treatment applies to a series of H and D curves each member of which is constructed on an intensity scale In this case the relation becomes

$$S = (1-p) \log \frac{t_1}{t_2},$$

where t_1 and t_2 are the constant exposure times for any two curves of the series Since S is seen to be independent of I, the curves must all be parallel, as in the above case of time-scale curves The equation shows in addition that they must move forward or backward parallel to the exposure axis, depending upon the value of p, as in the case treated above

It is not considered profitable to give, in this place, a review of the literature of the subject of the failure of the reciprocity law in photography As just stated, judging from the evidence it would seem that the results obtained in any investigation of this subject are to be considered as applicable only under the exact conditions prevailing as to emulsion, developer, and development conditions including also the method, kind, and intensity of illumination, and in addition, the size of the blackened areas There is, accordingly, a need of standardization This would not be necessary if the true laws of photographic action were understood That there is a need for standardization is clearly brought out in a study of the gradation wave-length effect which follows, in which similar discordances are found to prevail, and in which it is shown that it is questionable in astronomy to use laws of photographic action which have been obtained in the physical laboratory, unless more complete parallelism and standardization of conditions is secured than has prevailed in the past

It has been customary to assume the truth of Schwarzschild's blackening law in astronomical investigation Thus Tikhoff[1] finds an actual variation in Schwarzschild's constant

[1] Tikhoff G A Recherches nouvelles sur l absorption sélective et la diffusion de la lumiere dans les espaces interstellaires Compt rend 148 266 1909

exponent from 0.67 to 0.96, which is wrongly attributed to cosmic and not to its true photographic causes. J. A. Parkhurst[1] finds great variation in the Schwarzschild's constant, depending on intensity, which would be expected from Kron's law of photographic action. Reference has already been made to the work of Halm (p. 66).

Since the above was written, the results of L. A. Jones and E. Huse[2] on the subject of reciprocity failure have appeared, which can not be passed over without mention, for undoubtedly it is the most careful work on the subject since Kron's very comprehensive treatment. These authors find a notable departure from the reciprocity law, as well as from the law of Schwarzschild, their results confirming Kron in a general way in showing an optimal intensity, and in proving the wide variation in Schwarzschild's exponent p, depending on the intensity. However they differ markedly from Kron in finding a much broader region or range in intensity centering on the optimal intensity, within which the reciprocity law holds quite well, so that, on account of the flatness of the curve, the determination of the value of the optimal intensity from their data is subject to great uncertainty. The ranges in the values of p found by them, for three commonly used emulsions, and the corresponding intensity ranges, are as follows:

	p	Intensity Range
Seed 23	0.88 to 1.15	1 to 250,000
Seed 30	0.63 to 1.07	1 to 500,000
Cine positive	0.68 to 1.00	1 to 33,000

The highest illumination used was 131 meter-candles, the longest exposure time, 18.2 hours, the shortest, 0.00025 seconds, so that the time-range in practical work is well covered.

Application of Jones and Huse's data to astronomy. Since the amount and direction of the reciprocity failure depends, as just shown, on the intensity of the illumination falling on the photographic plate, it clearly is important to know the value of the latter, in the case of astronomical exposures, before these results can be utilized. The calculation will be made for the two cases of interest, focal images and extra-focal images, having an assumed diameter of 0.05 mm and 2.0 mm respectively, the diameter of the telescope objective being taken as

[1] Parkhurst, J. A., The evidence from photographic color filters in regard to the absorption of light in space. Astrophys. J. 30: 33. 1909.

[2] Jones, L. A., and Huse, E., On the relation between time and intensity in photographic exposure, J. Opt. Soc. Amer. 7: 1079. 1923.

20 inches Absorption and color index will be neglected The stellar brightness of a candle at a meter is assumed to be −14 18 magnitude The following table gives for the two cases the intensity of the illumination in meter-candles for stars ranging in brightness from that of Venus at maximum phase to the faintest which can be photographed with the 100-inch Mt Wilson reflector, computed from the following formula, which is easily derived

$$I = \frac{0\ 00000213R^2}{\text{antilog}\ 2/5\ M},$$

where M is the star's magnitude, and R is the ratio of diameter of objective to that of the photographic disc

Mag of star M	Extra-focal images m c	Focal images m c
− 4 0	5 30	8500 00
+ 1 0	0 053	85 0
6 0	0 00053	0 85
11 0	0 0000053	0 0085
16 0	0 000000053	0 000085
21 0	0 00000000053	0 00000085

The table is instructive, taken in connection with the data of Jones and Huse on the reciprocity failure They find little or no failure for illuminations between 0 1 m c and 100 m cs (1 c , p 1102) But it is seen from the table that the intensity of illumination in the case of most of the stars does not lie within these limits For example, in the case of extra-focal images, the illumination by a first magnitude star is only 0 053 m c , so that even for large telescopes the method of extra-focal images does not produce results free from reciprocity failure For focal images, the situation is better In this case, for the size of telescope chosen (20 ins), it is seen that the reciprocity law holds between the magnitudes 1 to 8 In the case of the 100-inch Mt Wilson reflector, the range would be 5 to 12 The lowest intensity used by Jones and Huse was 0 00025 m c , which is of the same order of magnitude as the intensity of the focal image of an 18th magnitude star in the 100-inch, which, from the formula given above, is 0 00034 m c While the size of the stellar disc which was chosen in the calculations (0 05 mm) may be underestimated in the case of the 100-inch telescope, the error is more than counterbalanced by the neglected factor of color index As a check, however, it can be computed from data given by

Jones and Huse (l c, p 1102), that at this low illumination level, a photographic density of 0 08, corresponding roughly with a threshold image in a telescope, is reached in an exposure time of 128 seconds, so that we should predict from this datum alone that an 18th magnitude star could be photographed by the 100-inch reflector with an exposure time of approximately two minutes, as is actually the case

The curves given by the authors showing the falling off in photographic density as the intensity of illumination decreases, are instructive in relation to the relative photographic efficiency of large and small telescopes For example, for a star of the 21st magnitude, the illumination of a focal image in the 100-inch reflector is 0 000022 m c Now this is ten times fainter than the lowest illumination used by Jones and Huse, for which the reciprocity failure should be very great Since, from their data, the failure increases with diminishing intensity, in contradiction to the law of Schwarzschild, it is clear that a large telescope gains in efficiency over a small one, photographically as well as optically, neutralizing the increased loss due to atmospheric unsteadiness and defects of following

Summing up, the indications are that the extent of the failure of the reciprocity law in astronomical photometry depends upon a number of factors, such as the star's magnitude, size of telescope, and upon the special circumstance of focal or extra-focal images This is in direct opposition to the law of Schwarzschild, which states that the failure is a constant under all conditions Additional factors of importance are brought out by the work of Kron, which indicates that development determines the zero point of the reciprocity failure (p 65)

HETERCHROMATIC ASTRONOMICAL PHOTOMETRY

Before considering the subject of heterchromatic astronomical photographic photometry its foundations should be briefly examined Since the density of a photographic deposit depends upon the kind of plate, the developer, light-intensity, exposure time, color of light, temperature and numerous other factors, the general equation may be written

$$D = \phi\,(I, t, a, b, c, d - - - -), \qquad (31)$$

in which *a*, *b*, *c*, *etc*, are parameters corresponding to all the above factors enumerated which have an effect upon the density The problem is to obtain D either graphically or analytically from an equation of this type In the preceding

section the various attempts which have been made to write an equation of this form have been reviewed A certain degree of success has been obtained with equations containing only three parameters, of the form

$$D = \phi\,(E, a, b, c),$$
$$E = I\ t, \tag{32}$$

and greater success when for E is substituted

$$E = \omega\,(I, t, u, v)\ \text{(Kron)} \tag{32a}$$

It is important to note that in any photometric investigation nearly all the parameters are necessarily neglected, through ignorance of their quantitative effect We are, therefore, driven to the artifice of attempting to keep them all constant, in which case equation 32a reduces to

$$D = \phi_1\,(I, t)$$

In practice such an equation is never written, the problem is actually solved by a graphical or interpolation method, which in principle is the same as deriving the equation Thus, for example, one plate only is used, so that, roughly speaking, emulsion and development parameters are kept constant Care extends quite naturally only to the conditions which are known to enter as factors The greater the knowledge of the photographic plate, the more numerous do the parameters become and the more difficult grows the problem While some parameters enter into photometry as accidental errors and can be eliminated by repeating, such as, for example, variations in sensitivity over the same plate, other errors enter in such a way that repeated observations do not "average out" the effect These are the systematic errors which are generally and rightly feared

In visual photometry the Purkinje phenomenon enters when lights of different colors are compared With the eye, the intensity rate of building up of visual sensation varies with color, being greater for red than for blue light A similar phenomenon was discovered to hold with regard to the photographic plate The phenomenon is well illustrated by the curves in Fig 25, which show the characteristic curves of a fast blue-sensitive plate obtained by the writer, which was exposed to monochromatic light of wave-lengths 4100A to 5200A The curves in Fig 26 are obtained by scaling from Fig 25 the values of gamma and maximum density In the next figure (Fig 27) similar gamma wave-length curves are

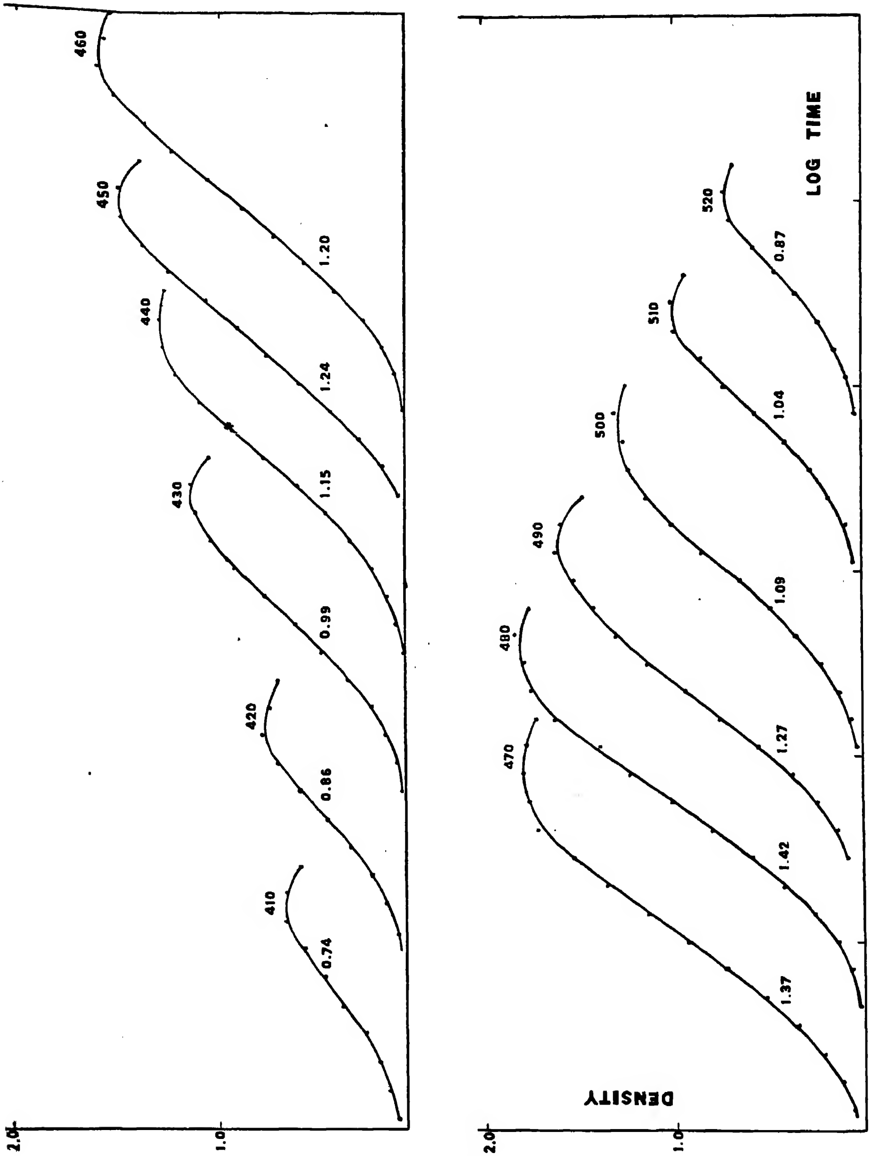


FIG. 25
Characteristic curves of a fast blue-sensitive plate

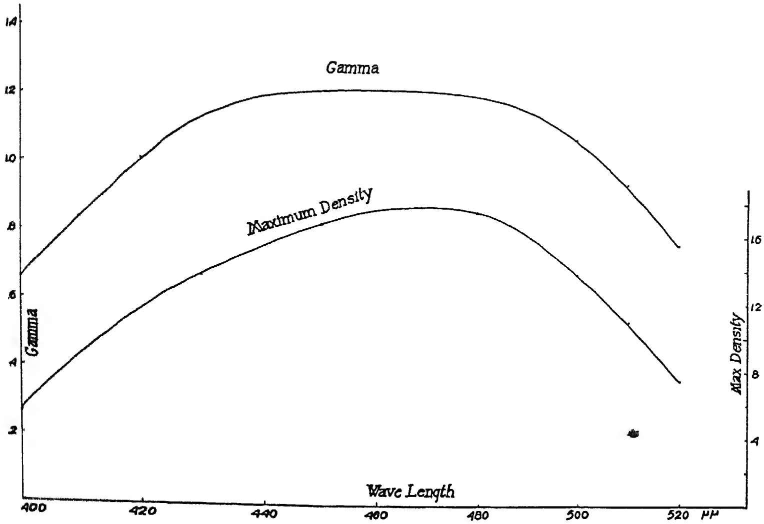


FIG 26
Curves for gamma and maximum density derived from Fig 25

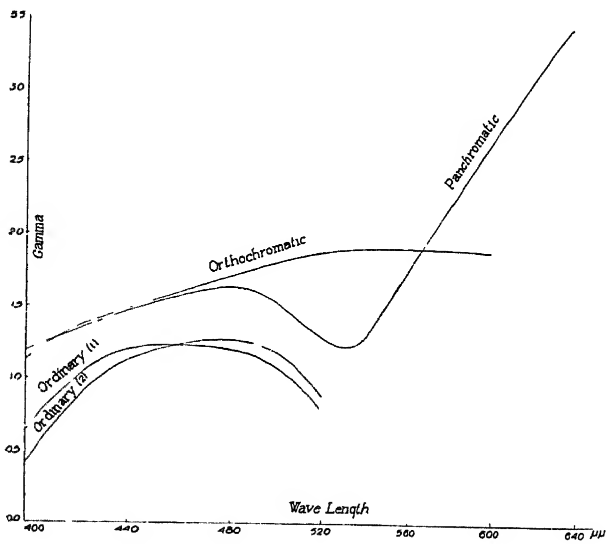


FIG 27
Variation of gamma with wave-length for various plates

plotted from results obtained from different types of emulsion which include an orthochromatic and panchromatic emulsion, as well as two ordinary or blue-sensitive emulsions. On the other hand T. Thorne Baker[1] finds no change of contrast with wave-length, for wave-lengths extending from the visible region down to 0.24 A., provided the exposure covers the correct range, and the development is complete.

Consider the general effect of variations in gamma or in astrogamma[2] with wave-length upon astronomical photographic photometry. Suppose a blue star S_b and a red star S_r, photographed on a panchromatic or red-sensitive plate, and suppose that for a given exposure time t, either the densities or the diameters of the two images are the same. Let another exposure be made of say ten times the intensity of the first. Since gamma is the rate of increase of density with exposure, and since from Fig. 27 gamma is greater for red light, it follows that the densities of the star images in the two cases will no longer be equal, that of the red star being now the greater. There is thus an indeterminateness in any comparison of stars of different colors. From the data of the curves it is not difficult to give a numerical measure to this indeterminateness in any given case. Thus, according to Hertzsprung[3] the effective wave-length of the average A-type star is 4266 A, with a change of 200 A for a change of unity in color-index. Consider two stars whose difference in color is represented by unity on the astronomical scale, one an A-type star, the other a K-type star, and suppose the out-of-focus photometric method is used, with "blackness" as the criterion for magnitude. From the gamma wave-length curve for ordinary emulsions (Fig. 26), the gammas are respectively 1.11 and 1.23 for the two stars considered. Let the photographic sensations or "blackness" be equal for S_a and S_k for an exposure-time t_o. Since

$$D = D_0 + \gamma \log t\,,$$

if the exposure-time is increased tenfold, it is clear that the density for the K-type star will now exceed that of the A-type by 0.12. Assuming the reciprocity law, the equation for the magnitude M is

$$D = D_0 + \frac{2}{5}\gamma M,$$

[1] Baker, T. T., The behaviour of silver bromide to rays of short wave length. Trans. Faraday Soc. **19**: 335. 1923.

[2] For definition of astrogamma, see p. 94.

[3] Hertzsprung, Effective wave lengths of 184 stars in the cluster N. G. C. 1647. *Astrophys. J.* **42**: 92. 1915.

from which

$$\delta M = 2\ 14\ \delta D$$

Since $\delta D = 0\ 12$, $\delta M = 0\ 26$, which is the amount the magnitude of the K-type star now nominally exceeds that of the A-type In other words, there is an uncertainty of 0 26 in the relative magnitude of the two stars, depending on the particular exposure-time chosen In general the difference is a linear function of the difference in the logarithms of the exposure-times and of the gammas corresponding to the effective wave-lengths It can be easily shown that

$$\delta M = 5 \frac{\gamma_B - \gamma_A}{\gamma_B + \gamma_A} \delta \log t, \tag{33}$$

where A and B refer to the two stars

There is evidence indicating that the low gamma and low maximum-density at λ 4100 and λ 5200 shown in Fig 25 are due partly to low light-intensity and low sensitivity, or more strictly to a low ratio of the two A corollary of great importance is that light of very feeble intensity produces a low photographic density which cannot be increased by increasing the exposure-time This is in agreement with H J Channon's[1] experimental results Channon attributes the phenomenon to the reversal action

If the effective wave-lengths of the stars all lie on the horizontal portion of the gamma wave-length curve, no ambiguity such as that which has just been calculated can arise The effect is in general due to the violet and ultra-violet light, and can obviously be decreased by using a suitable yellow filter It would seem that in common with the green and blue-green portions of the spectrum violet and ultra-violet radiation produce comparatively low contrasts or gammas and low maximum-densities in the majority of photographic emulsions, a matter of vital concern in accurate photometric work Gamma wave-length curves for all emulsions useful in astronomical photometry should be measured and filters matched with each in such a way as to make the effective radiations produce approximately equal gammas It will be noted in Fig 27 that the gamma wave-length curve of the orthochromatic emulsion is without the dip in the blue-green and is accordingly very valuable for photometric work This result, however, remains to be checked If emulsions can be found which give curves of this type they should prove exceedingly

[1] Channon H J Studies in photographic science Phot J 45 164 1920

useful in photometric work. In any case Figs. 27 and 28 conclusively show that orthochromatic and panchromatic plates are more suitable for accurate photometry than ordinary or blue-sensitive plates. Note the greater steepness of the curves throughout the violet in the case of the ordinary plates, a characteristic which impairs their usefulness for photometric work in a vitally important region of the spectrum.

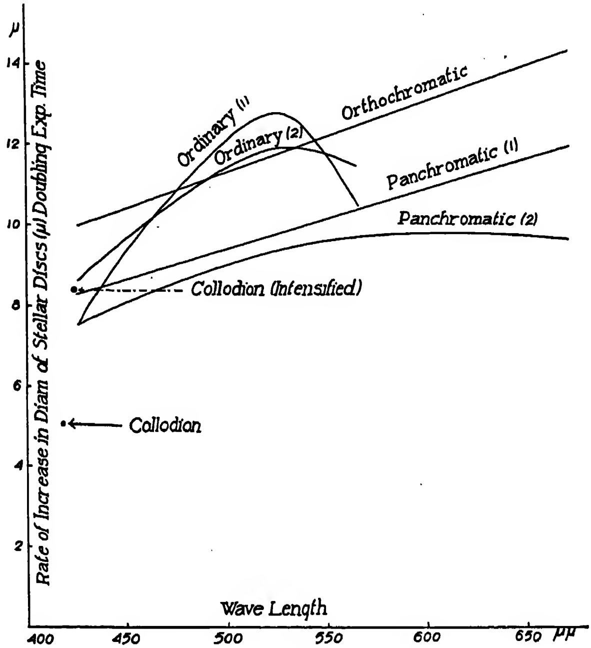
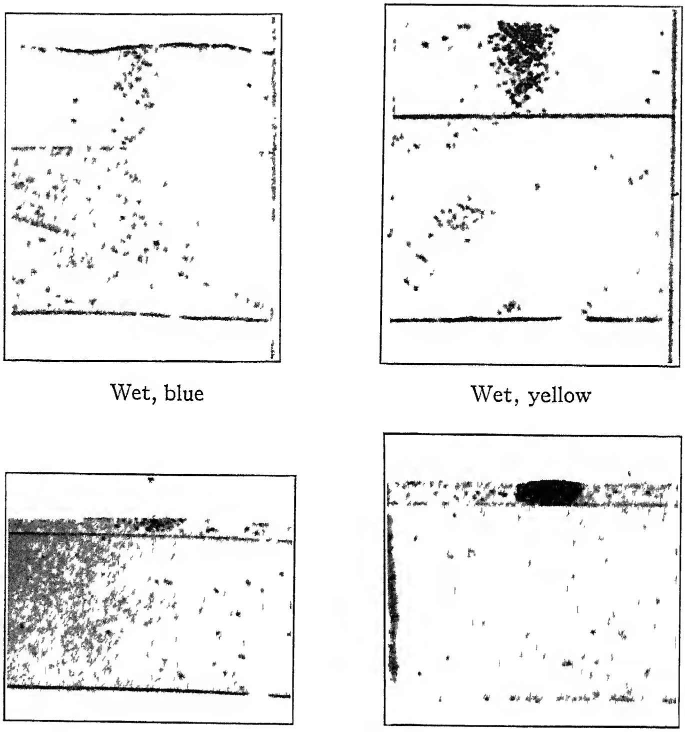


Fig. 28
Variation of Astrogamma with wave-length for various plates

That there is an enormous difference in the photographic action in star images for light of different colors is apparent from an examination of the images themselves. To show this the writer exposed an artificial star in a precision camera with blue and yellow filters interposed in turn. Orthochromatic film was used. For the moderately small diameter of image of 0.050 mm. the image of the blue star was exceedingly weak and gray, showing that it lay in a thin layer near the surface, while the image of the yellow star of the same diameter was very black. Sections were made of these star images which

are shown in Fig 29 The difference in penetration is to be noted



Wet, blue — Wet, yellow

Dry, blue — Dry, yellow

FIG 29

Magnified sections of stellar images

To show further the effect of color, sections were made of sensitometric strips which were obtained by exposure to monochromatic light of wave-lengths 4300A and 5600A, respectively As before, orthochromatic film was used In order to make the results comparable, equal densities were chosen The increase in penetration of the image for the longer wave-length is apparent (Fig 30)

In order to understand the actions here discussed and the reason for the change of gamma with wave-length, it is necessary to gain some idea of what contrast or gamma really is, and to apprehend its relation to some of the other physical

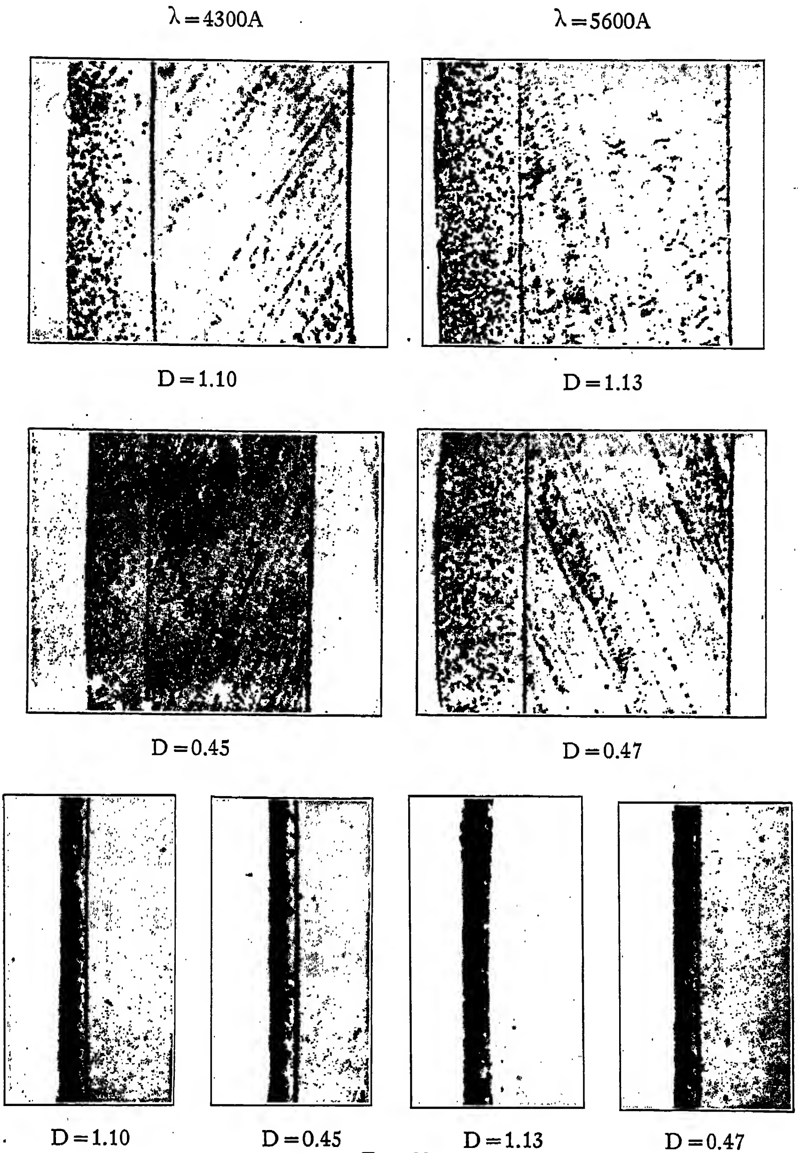


Fig. 30

Magnified sections of densitometric strips at two wave-lengths, wet and dry

constants of the emulsion Consider two normal exposures E, developing to a density D_1, $10E$, developing to a density D_2 In this case, by definition,

$$\gamma = D_2 - D_1$$

We also have (p 46),

$$M = c\,D,$$

where M is mass of reduced silver, and c the photometric constant

Consider now the wave-length changed to a value giving double the penetration into the film, other conditions remaining unchanged It is evident from the mass-equation above that the densities D_1 and D_2 will be doubled, so that the new γ will be

$$\gamma_1 = 2D_2 - 2D_1 = 2\gamma,$$

or γ is also doubled It is clear then that gamma is directly proportional to the penetration of the radiation into the film, other things being equal The absorption and scattering of a silver bromide emulsion in the ultra-violet is exceedingly strong, leading to low penetration and so to a low gamma In photographs of the spectrum the low value of gamma in the ultra-violet is very noticeable when comparing negatives made with short and long exposures With increasing wave-length the scattering and absorption are less The radiation accordingly penetrates to deeper layers of the emulsion, leading to increasing values of gamma

It is evident, however, from an inspection of Fig 27 that penetration is not the only factor governing gamma The dip in the curve in the green portion of the spectrum cannot be explained as being due to less penetration The sensitivity-curves show similar depressions at the same wave-lengths

It has been shown (p 55) that the characteristic curves of emulsions probably result from a certain normal action Let there be M silver grains per unit volume in the emulsion Let there be N grains per unit volume participating in the normal action For a thin layer the normal action is

$$dn = kI\,(N - n)\,dt,$$

where dn is the number of grains changed to latent image in the time dt, n the number changed up to the time t, k a constant for a particular plate and wave-length, I is light-intensity The reversal action is neglected

Many phenomena indicate that N is less than M, i e, all the grains in an emulsion do not participate in the normal action Consider for the moment a photoelectronic theory of the latent

image. Imagine with J. J. Thomson that the energy of the impinging radiation is in filaments, and in quanta as well, and therefore equivalent to a stream of material particles. Suppose that a collision of one quantum considered as an entity, or of a definite number of quanta, with one or a definite number of valency electrons in a silver halide grain is necessary and sufficient to make that grain developable. It follows at once from the laws of chance that a *constant fraction* kI of the grains uncollided with at any time t will suffer collisions in the next time interval dt and accordingly be made developable, which is exactly the law of normal action specified above.[1] This presupposes that all the grains N are in the same condition, or of the same sensitivity. Since the light quanta contain the frequency as a factor, a wave-length effect should be present, that is, N is a function of the wave-length. The statement above that the grains should be of the same sensitivity must be qualified. For emulsions of different sensitivities may be mixed and still normal action result. It is more accurate to state that grains are normal when they can be placed in one of a limited number of groups according to sensitivity, the extreme range of sensitivities being not too great, less than one to a hundred, say. It has been shown above (p. 59) that gamma decreases as the number of groups increases, and that the highest value of gamma is obtained when all the grains are of the same sensitivity.

Briefly then we can consider that an emulsion contains a certain amount of inert silver grains varying with wave-length between certain limits, which is dissolved out in the fixing bath. It was shown also that gamma contains the mass of silver as a factor. Since the inert silver disappears it does not figure in the effective mass. It is clear then that gamma and maximum-density must decrease the greater the amount of inert silver in the emulsion. It is this effect, depending on wave-length, combined with the penetration effect already considered, which gives the resultant curve of wave-length gamma. The phenomenon of reversal will doubtless be found to play a part. In the present state of the science of photography it is impossible to estimate the relative importance of the various factors.

Development undoubtedly plays a part in determining the mass of the inert grains. On account of differences in the reduction-potential of developers and differences in other properties as well, the latent image is not uniformly develop-

[1] This light-filament and quantum conception of Thomson's has been worked out in detail by Dr. L. Silberstein. See footnote p. 49.

able This is clearly brought out by differences in maximum-gamma and maximum-density obtained with developers of various types

It cannot be too strongly insisted that the true significance of measures and results obtained by photographic photometry in any particular case should be considered with reference to the particular problem being investigated In so far as the physics of the heavenly bodies depends upon photometric measures and scales, extreme circumspection is necessary in order that correct conclusions may be reached For example, in the case of faint stars requiring very long exposures, the shorter wave-lengths ($\lambda < 4400$ A) have a comparatively negligible effect upon the photographic plate (see Fig 25) This is due to the low penetration of the feeble short waves, leading to a low maximum density as just shown Given a faint star emitting short and long waves, the photographic effect of the short waves, as the exposure-time is increased, very soon reaches its peak and declines owing to reversal, while the effect due to the longer waves is still on the increase Although the application of a simple addition theorem to a case of this sort may be open to question, it seems certain that the proportional effects of the various wave-lengths are not constant for variations of time or intensity It would appear to be true from these considerations that for faint stars the short wave-lengths are comparatively without effect, leading to a classification of faint stars as redder than they actually are

A brief account of the results obtained by various investigators in studies of the important gradation wave-length effect will now be given The fact that photographic contrast changes with wave-length, giving rise to a photographic Purkinje effect, appears to have been discovered nearly simultaneously by Sir W Abney and J Precht[1] in 1899 Precht discovered the phenomenon through comparing sensitometric results obtained by using as sources of illumination a benzene lamp and an amylacetate lamp, the latter giving the yellower light He announced the action in the following form

(1) "Die relative chemische Helligkeit zweier Lichtquellen is abhangig von ihrer absoluten Intensitat,

(2) Die relative chemische Helligkeit zweier Lichtquellen is abhangig vom absoluten Wert der Expositionszeit "

Chapman-Jones[2] made a careful and detailed study of the phenomenon, many brands of plates of different degrees of

[1] Precht J Photographisches Analogon zum Phanomen von Purkinje, Arch wiss Photographie I , p 277 1899

[2] Chapman-Jones H , The effect of wave length on gradation Phot J 24 279 1900

color sensitivity being tested. He concluded that, in general, the gradation increases continuously from the ultra-violet to the red. An exception is found when development is in pyro-ammonia, in which case the gradation curve was found to be reversed. Typical characteristic or H. and D. curves for ultra-violet (B) and red illumination (A) are given in Fig. 31.

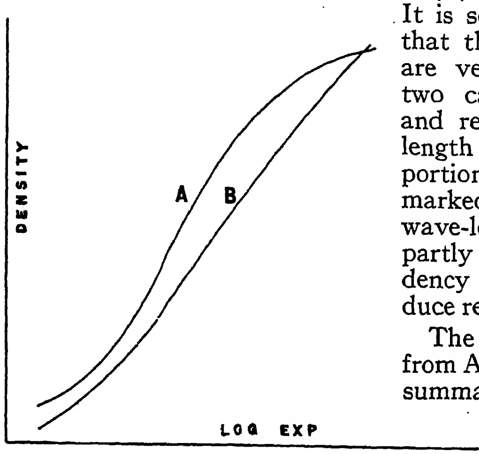


FIG. 31

Characteristic curves depending on wave-length

It is seen from the figure that the H. and D. curves are very different in the two cases of ultra-violet and red illumination, the length of the straight-line portion diminishing very markedly with increase of wave-length. This can be partly explained as a tendency of red light to produce reversal.

The following quotation from Abney's[1] paper clearly summarizes his conclusions. ". . . . Nearly two years ago, in an article in *Photography*, I indicated that a variation in gradation due to difference in the monochromatic light in which the exposure was made did exist, and some six months ago Chapman-Jones, in a paper communicated to the Royal Photographic Society, independently announced the same result from experiments made principally with orthochromatic plates with light passing through various colored media, and he generalized from his experiments that the smaller the wave-length the less steep was the gradation, the ultra-violet rays giving the least steep, and the red the most steep gradation. My experiments which at that time had been partially completed did not bear out this generalization to the full when pure silver salts were used, and my subsequent measurements with them show that the least steep gradation is that given by the monochromatic light to which the simple silver salt experimented with is most sensitive, and that the gradation becomes steeper as the wave-length of light employed departs in either direction in the spectrum from this point, the steepest grada-

[1] Abney, W. de W., On the variation in gradation of a developed photographic image when impressed by monochromatic light of different wave-lengths. Proc. Roy. Soc. 68: 300. 1901.

tion being given by the extreme red The case of orthochromatic plates, in which is a complex mixture of silver salt and dye, is necessarily less simple, involving consideration of the localities in the spectrum to which the dye or dyes, together with that of the silver salt, are most sensitive For this reason the simple salts have been experimented with in preference to the more complex organic compounds It may be thought that these results might be peculiar to the salt of silver experimented with A further series of experiments were conducted with the chloride of silver in gelatin The maximum sensitiveness of these plates was found to be near H[1] in the solar spectrum The gradation was found to be least at this point and increased when rays on each side of this point were employed to act on the film In the blue, near the F line, where the sensitiveness of the plate was very small, the gradation was excessively steep, as it also was in the extreme ultra-violet "

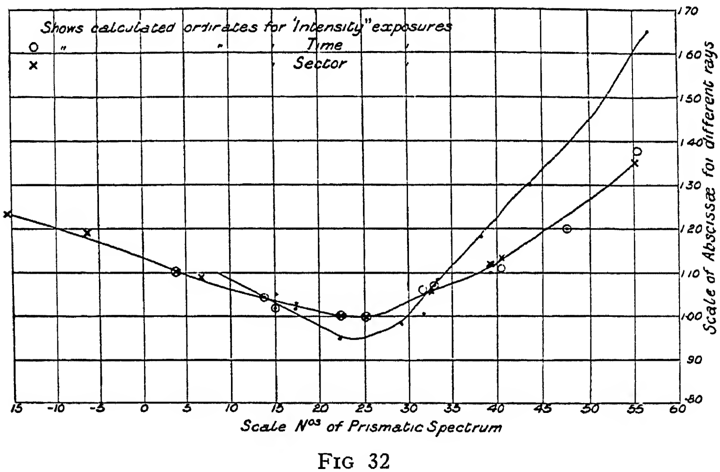


FIG 32
Abney's gradation curves

Abney's gradation curves are reproduced in Fig 32 The wave-length corresponding to Scale No 15 in the diagram is 372μμ, for No 20 is 452μμ, and for No 60 it is 673μμ

Abney's result that the least gradation is at the point of maximum sensitivity forms a beautiful and startling theorem Abney himself appears at a loss to account for it "What scientific explanation there is of this difference in true grada-

[1] λ = 3968 6 A

tion factor is hard to say." The following explanation suggests itself to the writer. It is shown (p. 58) that for emulsions composed of grains of one sensitivity, the gradation or gamma, is a maximum, and that as the number of groups of grains of varying sensitivity increases, gamma decreases. In the case of the emulsion studied by Abney, it is possible that at the wave-length of maximum sensitivity a greater number of grain groups come into action, leading, therefore, to a lower gamma. That it is not a general phenomenon, however, appears to be conclusively shown by the experiments of Chapman-Jones and those of the writer previously described.

Contrary to all results quoted so far, G. Leimbach[1] obtained no change of gradation with wave-length: "Im Bereiche der normalen Belichtung wächst die Schwärzung mit zunehmender Belichtungszeit nach demselben Gesetz, oder: *Die Gradation ist für alle Wellenlängen dieselbe*" (p. 188).

As already remarked, the subject is of the greatest importance astronomically. For example, Tikhoff[2] concluded from a photographic study in which colored filters were used that light suffers an appreciable loss in its passage through space. This conclusion is questioned by J. A. Parkhurst[3] and by H. E. Ives[4] from studies of the behavior of the plate itself. Parkhurst concludes: " . . . this experiment with the photographic color-filter furnishes no evidence either for or against an absorption or scattering of light in space." As Ives shows, Tikhoff's conclusion is based upon the assumption that the gradation of the photographic plate is the same for all colors. Ives' paper on the subject is of importance in indicating causes underlying the phenomenon and in giving an explanation of the contradictions found by different investigators. He concludes that the factors of importance in determining the variation of gradation with wave-length are: (1) penetration of the dye in the film; (2) penetration of the light; (3) penetration of the developer. He notes the important fact that when plates are color sensitized by bathing, the penetration of the dye into the emulsion is superficial. In the case of plates color sensitized by the manufacturer, the dye is incorporated in the emulsion before coating so that the light sensitive layer is much thicker and consequently a much higher

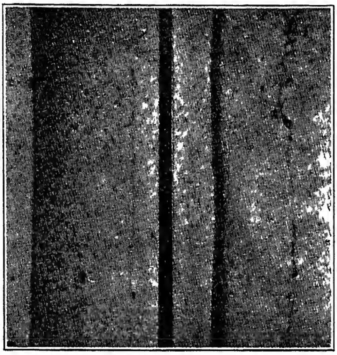
[1] Leimbach, G., Die absolute Strahlungsempfindlichkeit von Bromsilbergelatine platten gegen Licht verschiedener Wellenlänge. Zeits. wiss. Phot. **7**: 181. 1909.

[2] Tikhoff, G. A., Recherches nouvelles sur l'absorption sélective et la diffusion de la lumière dans les espaces interstellaires. Comp. Rend. **148**: 266. 1909.

[3] Parkhurst, J. A., A property of the photographic plate analogous to the Purjinke effect. Astrophys. J. **49**: 202. 1919.

[4] Ives, H. E., Some photographic phenomena bearing upon dispersion of light in space. Astrophys. J. **31**: 157. 1910.

gradation is obtained "With a Cramer Instantaneous Isochromatic plate, the red gradation was steeper than the blue With a Seed 26 plate, bathed for two minutes in a 1 100,000 solution of pinacyanol the blue gradation was steeper than the red " Ives suggests that Leimbach's negative results were due to surface development, since he used ferrous oxalate developer for short development time only Fig 33 is a reproduction of photomicrograms of film sections made by Ives, *a* shows the penetration of the action in the case of exposure to blue light, *b* the same for red light, the film being red-sensitized by bathing



a *b*
FIG 33
Photomicrograms of film sections by Ives

Parkhurst[1] has sought to determine directly the change of gradation of the photographic plate for stars of different classes He finds a difference of 1 5 per cent in the direction of higher gradation for the redder star class, but concludes "We, therefore, reach the conclusion that for Seed 27 plates and extra focal images the slight systematic effect which may be present is nearly or quite masked by the accidental errors arising principally from the lack of uniformity in the film "

No attempt has been made in the above review to present all the data and investigations bearing on the very important subject of the dependence of gradation upon wave-length The contradictions and confusion are patent without any further citation of cases Careful study of the methods employed by the various investigators does not in general disclose any lack of care in details or narrowness in treatment Without any pretense to being exhaustive, many of the investigations were admirably planned and carefully made

Assuming as we must that no flagrant errors were made by any or all of the investigators, the only supposition which can be made, in an attempt to account for the discrepancies, is that the wave-length effect upon gradation, just as the wave-length effect upon sensitivity, is strongly variant with the emulsion, and, to a certain extent, with the developer and development conditions It can not be said that this possi-

[1] Parkhurst J A A property of the photographic plate analogous to the Purkinje effect Astrophys J **49** 202 1919

bility was not present in the minds of many of the investigators, for, in several cases, a large number of plates of various kinds, including blue-sensitive, isochromatic and panchromatic, were used in their measurements Also, though in a narrower way, more than one developer was in general used Notwithstanding this, no alternative is seen to the following *the gradation wave-length curve is sensitive to emulsion, developer, and development conditions, and accordingly, can not be generalized*

Consider first the effect of the emulsion characteristics upon variations of gradation There are several factors of importance which will be enumerated briefly First, there is the opacity factor, which is operative, as shown on page 55, but concerning which little or nothing is actually known, secondly, the varying sensitivity of the different groups of grains in the emulsion, which indirectly affects gamma or gradation, as shown on page 55, thirdly, there is the so-called "reciprocity-effect" or failure of the reciprocity law This is operative in two ways (*a*) through the emulsion, the various groups of grains in the emulsion responding in different degrees to light of different colors, (*b*) through variations in the intensity and quality of the illumination used in obtaining the gradation characteristics sought The illumination naturally varied considerably among the various experiments, so that variations and even reversal of the gradation curve might be expected depending upon the particular light source chosen That this is true would follow directly from Kron's law of light action (p 63) Also, Parkhurst[1] has found that the exponent p in the Schwarzschild equation varies with the intensity as well as with the color of the incident light

Concerning the effect of developer upon gradation but little is actually known Referring to page 80 and to Ives' work (p 85) it will be seen that gradation depends upon the depth of the layer of the emulsion which is developed Ives (p 86) explains Leimbach's results as due to an unusually shallow development Also, Chapman-Jones (p 82) obtains a complete reversal of the gradation wave-length curve when pyro-ammonia is used as a developer Just how this last acts is not even guessed at Looking at the action in a general way, a helpful point of view is furnished by a consideration of the action of reducers We may from this standpoint view depth-development as a process depending upon the diffusion characteristics of a developer combined with its reaction or devel-

[1] Parkhurst, J A, The evidence from photographic color filters in regard to the absorption of light in space Astrophys J 30 33 1909

opment velocity, and, in addition, its tanning characteristics, in which the reaction products of development and tanning action play the part of inhibitors or retarders It can not be said that this is merely speculation, for the Eberhard[2] effect and the Kostinsky effect (p 183) require for their explanation similar chemical actions Eberhard has shown that the density of the photographic deposit depends upon its area With the above assumption this is explicable It is evident that the developer and development conditions are of great importance in a study of the gradation effect and must be carefully taken into consideration in any future studies of the subject

The Eberhard effect mentioned above is another possible cause of the divergences described It is quite evident that with this effect present the gradation wave-length characteristics of emulsions obtained by astronomers from studies of small images either in focus or out of focus may be very different from physical laboratory determinations based upon measurements on extended areas of the order of one hundred times larger Not only is the Eberhard effect of importance in this connection but it has a serious effect as well upon monochromatic and white light photometry, so that in these fields also astronomers must apply with caution and reservation results obtained in the physical laboratory

ASTRONOMICAL PHOTOGRAPHIC PHOTOMETRY DEPENDING ON DIAMETER OF IMAGES

This subject will be developed in some detail A brief historical review of the development of the method should be given Simultaneously with the first successful photograph of the stars, by Bond in 1850, made on a wet collodion plate, a method of astronomical photographic photometry was developed, based upon the fact at once evident from an inspection of the plates, that the size or diameter of the small disk images formed by the light of the stars individually increased in a progressive manner with the brightness of the star, or its magnitude Correlating all his data, Bond found the following relationship

$$y^2 = Q + Pt, \tag{34}$$

where y is the diameter of the star image, t the exposure time, and Q and P constants The intensity I can replace t in this equation, from the reciprocity relation In order to see how

[2] Eberhard G Ueber die gegenseitige Beeinflussung benachbarter Felder auf einer Bromsilberplatte Physik Zeit 13 288 1912

closely such a formula fits results obtained with modern dry plates, the writer impressed upon a Seed 23 plate a series of artificial star images with increasing exposure times. The results are plotted in Fig. 34 according to Bond's formula.

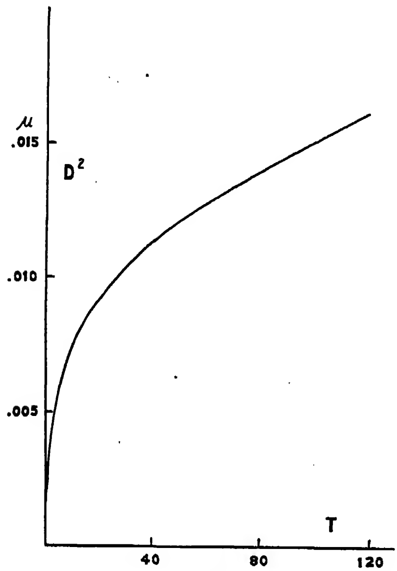


FIG. 34
Bond's formula compared with observation

It is seen that the linear relation between y^2 and t is far from being realized except for a very short range of 1 to 4 (about 1.5 magnitudes) near the beginning of the curve. Bond noted that the sensitiveness of the plate increased toward the end of the exposure, a fact which appears capable of explaining his comparatively unique formula.

The first investigation of the relation between image size, and brightness and exposure time for the modern dry plate, was made by E. C. Pickering in 1886.[1] In his investigation, brightness and exposure time appear correctly as logarithmic functions. His formula is involved, however, and contains numerous variables of uncertain value in consequence of which it does not appear to be of practical use.

In 1889, J. Scheiner, from an investigation of the relative diameters of images of a group of stars in the Pleiades, checked by measures on artificial stars in the laboratory, deduced a relation equivalent to

$$d = a + b \log I, \tag{35}$$

where d = diameter of image, a and b are constants, and I = brightness. The total range of intensity upon which the formula is based is about one to four hundred. Scheiner also investigated the relation with time substituted for brightness, for which he found a different type of relationship,

$$d = d_o \sqrt{t}\,. \tag{35a}$$

[1] Pickering, E. C., An investigation in stellar photometry. Memoirs Amer. Acad. Sci. 11: 179 1886

This formula, however, was not substantiated by exposures to the Pleiades In the same year C L V Charlier found from an exhaustive investigation

$$d = p\,I^{a}, \quad d = d^{o}\sqrt[4]{t}, \tag{36}$$

where a, p, and d_o are constants to be determined from the observations In order to see how formulae involving some root of the exposure time, as in Scheiner's and Charlier's formulae, fit modern data, the results obtained by the writer on Seed 23 plates, previously quoted, were plotted against the second, fourth, sixth and eighth root of the exposure time The results are shown in Fig 35 It will be noticed that as the root becomes higher the relation becomes more nearly linear if the smaller measured diameters are excluded If the lenses used were of poor quality and other conditions were such that it was not possible to obtain small star images such as are obtained at the present day, it is not surprising that equations of this type were obtained In recent work on the subject, along more accurate lines, investigators have introduced logarithmic functions of time and intensity into their equation instead of these quantities themselves and their roots This procedure is based upon the principle that equal fractional or percentage variations in the exposure produce equal effects on the photographic plate, at least in its useful exposure region Scheiner's formula 35 is of this type This

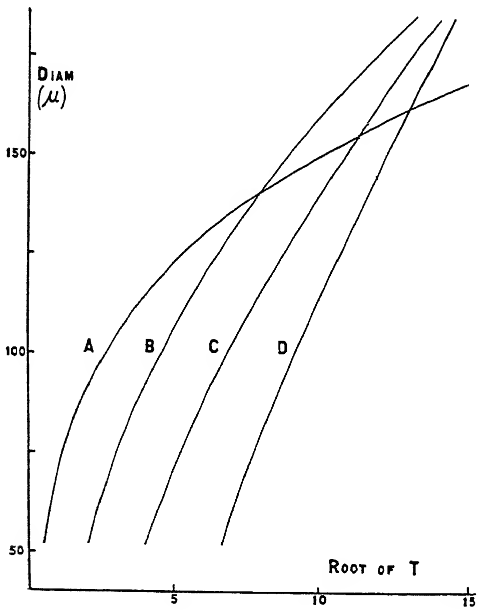


FIG 35
Diameters of images plotted against roots of exposure times

formula has been verified by C E K Mees[1] in a series of experiments with slit images of various sizes in which the range of exposure time was one to five hundred Astronomical observations in general, however, appear to be better satisfied by a modification of this formulae, the Greenwich equation,

$$d^{1/2} = a + b \log I, \tag{37}$$

in which the diameter is replaced by its square root In Fig 36 are plotted the diameters of the artificial star images previously mentioned, according to each of these equations

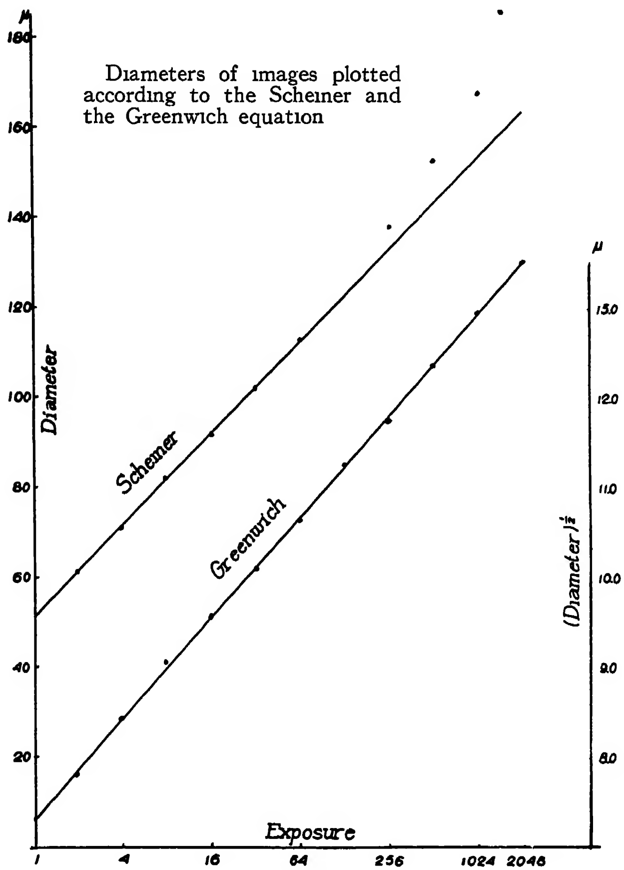


Fig 36

[1] Mees C E K On the relation between the diameter of the photographic image of a point and the exposure which produced it Astrophys J 33 81 1911

In order to investigate the applicability of the various formulae when very small images, less than about 50μ, are included within the range of observation, the following series of diameters of artificial star images made by the writer was chosen

TABLE 8

Exposure Time	Measured Diameter	Diameter Greenwich Formula	Error of Formula
(seconds)	(μ)	(μ)	(μ)
1 0	19	30	+11
1 9	29	36	+ 7
3 8	39	43	+ 4
7 5	47	51	+ 4
15 0	57	58	+ 1
30 0	68	67	− 1
60 0	77	77	0
120 0	86	86	0
240 0	97	97	0
480 0	109	108	− 1
960 0	120	120	0
1920 0	130	131	+ 1

This series is plotted in Fig 37 according to the Scheiner equation 35, and the Greenwich equation 37 The Scheiner

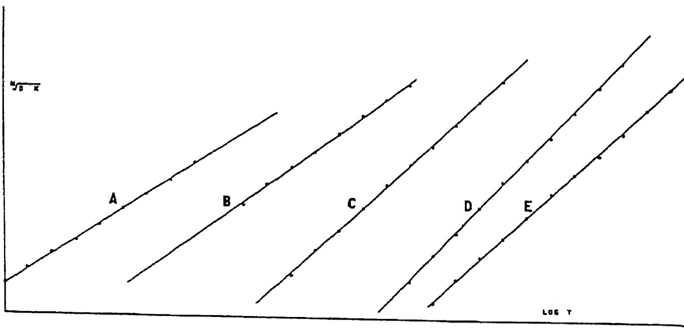


FIG 37
A, Scheiner, B, Greenwich, C, D, E, new formulae

equation fits the observations, except in the case of the larger diameters, as was to be expected On the other hand the Greenwich equation is seen to be very seriously in error in the region of the smaller diameters It is not to be wondered at that this error is not generally evident, for in the case of most astronomical photometric observations, the smallest images obtainable with large telescopes are about 50μ in diameter

With the very largest telescopes, they are no smaller than about 80μ. However, in the case of short-focus telescopes which are highly corrected, under the best atmospheric conditions, photographic images as small as, or smaller than 25μ, are usually obtained, so that in cases of this sort the error in the Greenwich formula should be at once manifest. The modification which it would be necessary to make in the formula to make it fit the smaller diameters as well as the larger is quite evident. It is only necessary to add a constant h to the diameter, so that the equation becomes:

$$\sqrt{d + h} = a + b \log I, \tag{38}$$

or, in magnitudes,

$$\sqrt{d + h} = a' + b'm. \tag{38a}$$

Curves C, D, and E (Fig. 37) show the result when h is 50, 100, and 150, respectively. In the latter case (h = 150) the representation is entirely satisfactory. The correct value of h in any given case can be sufficiently well determined in a few minutes by graphical methods. The use of the Greenwich equation in astronomical photometry has doubtless lead to errors of a systematic kind.

The problem of image growth will now be treated mathematically in so far as this is practicable. It was formerly considered that the growth of images was a chemical phenomenon, grains of silver bromide being supposed to develop by infection from neighboring grains. While there may be a certain measure of truth in this, in so far as secondary actions are concerned, it is not necessary to resort to hypotheses of this nature for all the evidence tends to show that there is a distribution of scattered light in the region immediately surrounding an image which can account for its spread or increase in size, as will be seen. It is quite impossible to deduce mathematically the curve of light distribution in a direct manner from the causative side for it has already been pointed out how numerous are the separate factors which enter into the problem, very few of which can be expressed in any kind of mathematical form. Any attempts at solving the problem must, therefore, be confined to the other end, *i.e.*, working backward from photographic effect to light distribution.

Starting from the simplest assumption, suppose that the light distribution is exponential, following Bouguer's law of intensity variation in a simple absorbing medium. In that case

$$I = I_o e^{-Kx}, \tag{39}$$

where I is the specific light intensity at a distance x measured perpendicularly from the edge of an image having either a straight or circular boundary. I_o is the intensity just within the boundary, κ is a parameter to be determined. κ depends on the turbidity of the emulsion and on other factors which have already been discussed in some detail (p. 38). At the edge of any photographic image, there is a photographic density d_o of small but definite value. If a series of images are impressed on the plate with a constant exposure time, but varying intensity, it follows that the light intensity, i, producing the density d_o must be the same for all the images. Accordingly, if the central intensity I is increased to I', the increase in size of the image is governed by the condition $i =$ constant. We have then, applying equation (39) to the border of the image,

$$i = Ie^{-\kappa x} = \text{constant} = c''', \tag{40}$$

or, $\log I - \kappa x = c''$,

and $$x = c' + \frac{1}{\kappa} \log I \tag{40a}$$

Applying the equation to measures of star images whose total diameter is d,

$$d = g + 2x,$$

where g is the diameter of the disk of supposed uniform illumination. Substituting d gives

$$d = c + \frac{2}{\kappa} \log I \tag{41}$$

Comparing this with (35), Scheiner's equation, it appears that the latter is a mathematical consequence of a law of light distribution given by Bouguer's law. From the fact that Scheiner's equation is found to be true for a long range of exposure values, it follows that within the corresponding range of distance in the plate ($20\mu\pm$ to $150\mu\pm$), Bouguer's simple law applies.

The term astrogamma, Γ, is proposed by the writer to designate the coefficient of log E in the Scheiner equation, so that

$$\text{Diameter} = a + \Gamma \log_{10} E, \tag{42}$$

which is suggestive of its relation to gamma (γ), the latter having been defined as the coefficient of log E in the cor-

responding equation involving photographic density, namely,

$$\text{Density} = a + \gamma \log_{10} E \tag{43}$$

Neither equation is of universal applicability. Equation (43) holds only for a limited portion of the H and D curve, or the straight-line portion, its error being very large near threshold and for overexposures. Equation (42) is of wider applicability, as it appears to be accurate even to the threshold point (see Fig. 37) but fails for the higher exposures.

If the law of light distribution given by

$$i = Ie^{-\kappa\left(\sqrt{x+\frac{g}{2}} - \sqrt{\frac{g}{2}}\right)} \tag{44}$$

is assumed a similar process of reasoning leads to the equation

$$\sqrt{x+\frac{g}{2}} = c' + \frac{1}{\kappa} \log_e I,$$

or in terms of diameters,

$$d = c + \frac{\sqrt{2}}{\kappa} \log_e I, \tag{44a}$$

which is the Greenwich equation. Assuming again a distribution given by

$$i = Ie^{-\kappa\left(\sqrt{x+\frac{g}{2}+q} - \sqrt{\frac{g}{2}+q}\right)}, \tag{45}$$

we obtain

$$\sqrt{d+2q} = c + \frac{\sqrt{2}}{\kappa} \log_e I \tag{45a}$$

Putting $2q = h$, it is seen that (45a) is identical with the empirical equation (38) which has been found to represent best the diameters of star images, from threshold to maximum.

The curves of light distribution for the three cases of the Scheiner, Greenwich, and new formula, computed from (40), (44) and (45) for several values of g, are given in Figs. 38 and 39. Fig. 39 is for the region 50μ to 100μ. The curves clearly bring out the characteristic differences of the formulae. For example, the Scheiner and new formula agree near the edge of the image as was to be expected, but diverge widely at greater distances. The Greenwich formula appears to stand alone at all distances but in the region of greater distances this divergence is only apparent, for if in Fig. 39 the ordinates of the Greenwich formula C are multiplied by 5.9, agreement with the ordinates for the new formula B is obtained.

Cause of the Deviation from the Scheiner Equation Experiments have been made for the purpose of throwing light on the reasons for the increased rate of growth of images with increased exposure They are, however, not sufficiently far advanced to be made use of at the present time The data obtained by Bloch and Renwick (p 39) on the opacity of emulsions are suggestive and useful in the present connection According to their experiments, the falling off in light intensity downward in a film is less rapid in the lower layers than in the upper, which is entirely parallel with the change of rate which has been found in the direction perpendicular to this and now being studied Bloch and Renwick's equation is

$$D = 0\ 66\ W^{0\ 64} \text{ (white light)} \tag{46}$$

Putting x = distance below the surface,

$$W = a\ x,$$

also,

$$D = \log \frac{Io}{I},$$

where I is the light intensity at the depth x Substitution gives

$$I = I_0\ e^{-\kappa x^{0\ 64}} \tag{47}$$

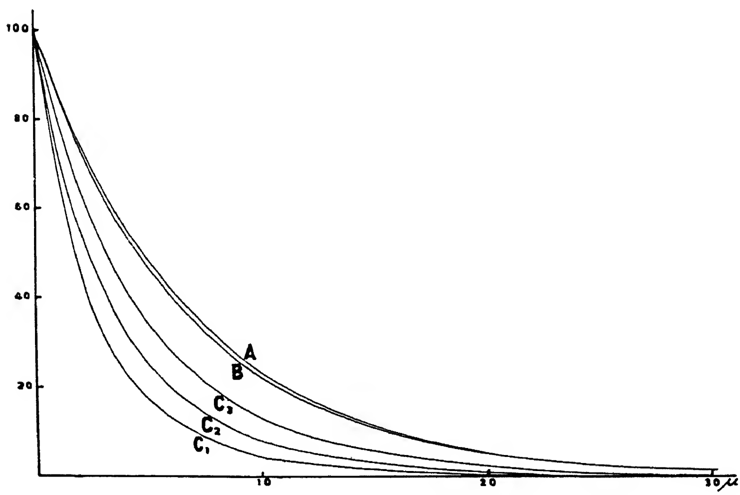


FIG 38
Light distribution near edge of image

This is analogous to the theoretical light distribution corresponding to the Greenwich equation (p. 95), the only difference being in the exponent of x, which is 0.64 instead of 0.50, g being assumed zero. Thus there appears to be a similarity between the light distributions downward and laterally in a photographic emulsion.

Consider two round images of equal intensity on the photographic plate, whose diameters are respectively of the order .05 mm. and 1 mm. If all the physical constants of the emulsion are known the light distribution in the two cases is mathematically determinable. This problem appears never to have been attacked. It is easy to see, nevertheless, that in a general way the light distribution should not be the same for the two images, such as would be required by Bouguer's law. The falling off in light intensity in the case of the larger image should be less rapid, on account of re-enforcing illumination filtering out from the central region of the image. The same reasoning applies to a certain extent in the case of two true star images formed by stars very unequal in magnitude, and is materially strengthened when expansion of the disks caused by atmospheric unsteadiness is considered. Accordingly, with increase in intensity, or increase in exposure time, we

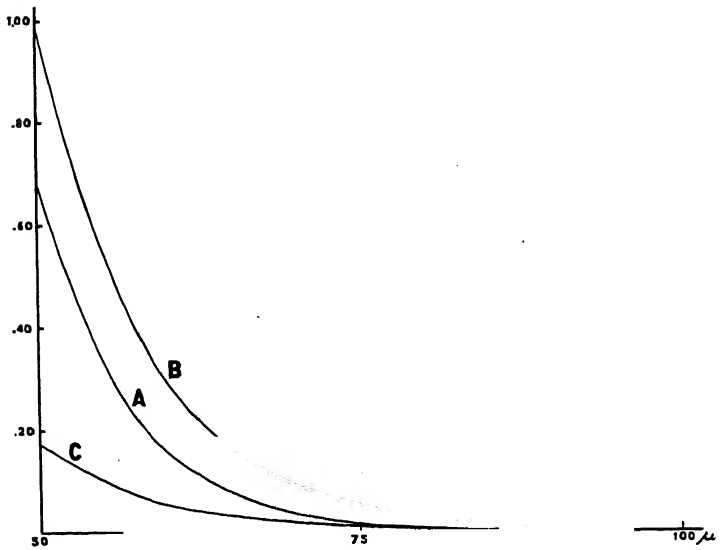


FIG. 39
Light distribution at some distance from edge of image

should expect an increase in the rate of growth such as is tacitly contained in the Greenwich and in the new equation But the fact that the Greenwich equation is greatly in error for the smallest diameters must be considered It should be noted that Bloch and Renwick's equation was not based on data from extremely thin emulsions, so that there is no experi-

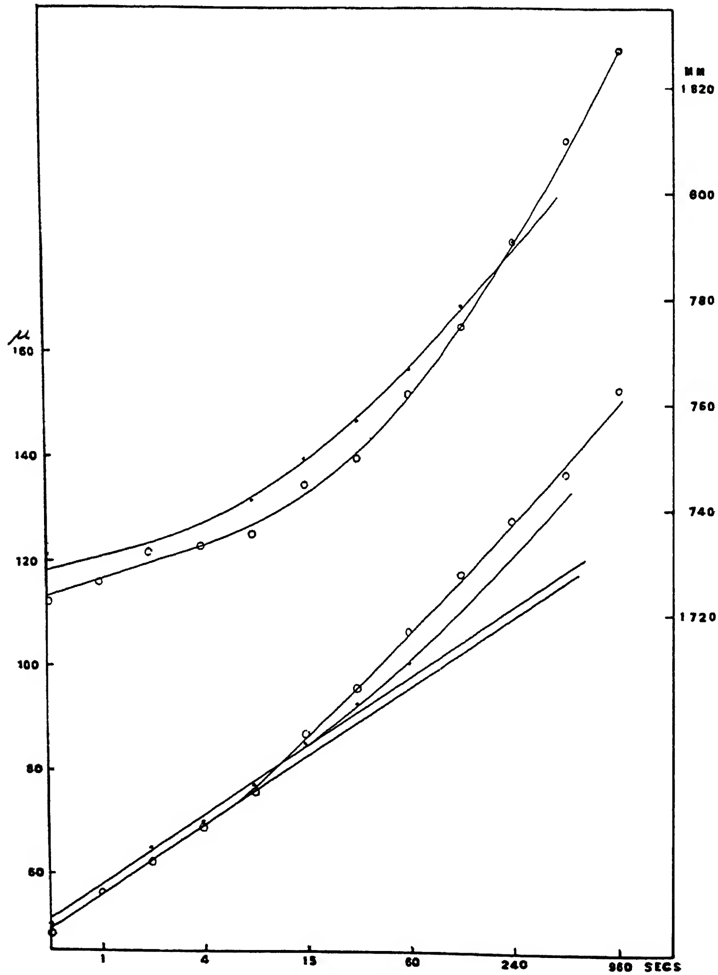


Fig 40
Turbidity for large and small images

mental evidence that it actually holds for the top layers of an emulsion. Its applicability, as embodied in the Greenwich equation, to the region immediately bordering the true edge of an image is accordingly open to question.

In order to determine experimentally the relative turbidity of large and small images exposures were made in the precision camera. (For definition of turbidity, Δ, see p. 105.) Two developers were employed, leading to the curves formed by the points and circles. The measured diameters are shown in Fig. 40 along with the corresponding curves just mentioned. Neglecting the lower portion of the curves for the large image, which region is affected by gelatin contraction (p. 163), it is seen that the turbidity is greater in the case of the large image than for the small. This is confirmed in the case of small images of much less difference in size, in which the gelatin contraction is less prominent, as the following shows. The geometrical diameter is computed from the measured size of holes in the test-object. The plate was Seed 23.

TABLE 9

Illumination: white light (no color filter used)

Aperture	Diam. of Image (Geometrical) mm.	Diam. of Image (Initial) mm.	Δ μ
1	.004	.010	9.0
2	.006	.012	9.2
3	.008	.015	9.5
4	.011	.018	9.7
5	.014	.020	9.9
6	.017	.031	10.8
7	.025	.046	11.2
8	.034	.058	11.5
9	.057	.085	11.9

In the case of the images of the five smallest apertures, no departure from Scheiner's law was apparent over an exposure range of 1 second to 32 minutes or 1 to 1920. For the largest aperture, No. 9, a strictly linear relation (Scheiner's) held over a range of 1 second to 8 minutes or 1 to 480. Using the same test-object and reducing the intensity of the light 50 times diminished Δ about ten per cent for the larger holes, but had no effect on Δ for the smaller apertures. In addition it was found for this case of diminished light-intensity that there was neither increase or decrease of the range over which the linear relation holds.

In order to obtain results for a very intense diffraction image the diffusing screen between light and test-object was removed entirely and the light adjusted in position until it was in line with aperture No 1 and the center of the lens With this set-up an exposure of 1 second gave an image having a diameter of 64μ, which is as large as was previously obtained with a 6 minute exposure through this same aperture From these results it can be calculated that the increase in intensity is 300 times (6 2 magnitudes) The value of △ was found to increase from its old value 9 0 to 12 5, or to approximately the value for aperture 9 (Table 9) However, the shape of the curve was quite different, the linear relation between diameter and exposure time holding only over a range from 1 second to 30 seconds Since in this experiment there has been no change in the size of the geometrical image, nor in the character of the aberration or diffraction pattern, the conditions are analogous to the practical astronomical case of photographing a field of stars of varying brightness The conclusion is, that under these circumstances the deduced relative magnitudes are dependent on the exposure time, △ being a function of the magnitude of the stars This phenomenon is similar to the Purkinje or color-gradation change In accurate astronomical photometry it must not be overlooked

The dependence of △ upon the telescope as well as upon the kind of plate used is shown in results now to be given which are summarized from material kindly furnished by Dr J A Parkhurst of the Yerkes Observatory and which were taken from a thesis of Miss Alice Farnsworth

40-Inch Refractor The focal length is 57 feet Size of minimum image (threshold) approximately 0 080 mm There does not appear to be any straight-line portion to the diameter-magnitude curve The following are the values of △, using Cramer Iso plates

Diameter mm	△ mm
0 080	0 020
0 500	0 052

12-Inch Reflector The focal length is 93 inches Size of minimum image 0 017 mm In the case of this telescope the relation between diameter and magnitude, as well as diameter and log exposure-time appears to be linear over the unusually wide range of approximately 8 magnitudes The deduced values of △ are

Seed 30 plates	0 011 mm
Cramer Iso plates	0 020 mm

Ultra-violet 6-inch Camera. The focal length is 32 inches. Size of minimum image .014 mm. The characteristics are the same as those of the reflector noted above. The values of Δ are:

Seed 30 plates	0.013 mm.
Cramer Iso plates	0.020 mm.

Turbidity of Fine-Grain Plates. The following table shows results obtained for turbidity of Seed Lantern and yellow dyed Seed Lantern plates. The same test-object was used as before.

TABLE 10

TURBIDITY

		Seed Lantern		Yellow Dyed Seed Lantern	
Aperture	Geometrical Diameter of Image	Threshold Diameter of Image	Δ μ	Threshold Diameter of Image	Δ μ
1	.004 *mm*	.007 *mm*	8.0	.005 *mm*	...
2	.006	.007	8.2	.005	...
3	.008	.007	8.5	.006	...
4	.011	.009	9.0	.008	...
5	.014	.011	10.4	.010	...
6	.017	.018	12.1	.014	5.1
7	.025	.021	12.3	.020	6.5
8	.034	.030	11.5	.025	6.9
9	.057	.056	11.6	.053	8.2

A considerable decrease in turbidity for yellow dyed plates is apparent. On the other hand, we obtain the rather unexpected result that there is no difference in the turbidity of Seed 23 (Table 9) and Seed Lantern.

It is of interest to compare in the above table the second and fifth columns. Except for the smallest aperture, the threshold diameter is seen to be less than the calculated geometrical diameter. In the case of Seed 23, Table 9, the reverse is true. Since the undyed Seed Lantern plate shows smaller threshold images than Seed 23, although of the same turbidity, threshold size cannot be governed solely by turbidity. It appears quite likely that it is correlated with sharpness and therefore proportional to turbidity divided by gamma (p. 116).

Relative Turbidity of Star Images and Spectral Lines. In order to compare relative turbidities in this case, it is evident from the preceding remarks that the images must be approximately of the same width. A test-object was made having a

slit aperture about 1 mm in width, with a circular hole on each side of the same diameter The mean results are as follows, the plate being Seed 23

	Star Image	Slit
Width of image, 1 sec exposure	0 082 mm	0 068 mm
Observed Δ, exposure range 1 sec - 120 sec	8 2μ	8 2μ

A violet and aesculine filter was used, which accounts for the small value of Δ as compared with that in Table 9, in which white light was used The values of Δ for star image and slit are seen to be the same This is contrary to expectation, for it was supposed that the slit would show the greatest turbidity, in other words, at a given small distance from the edge of the diffraction image of star and line, the illumination should be greater in the case of the line image It is possible that gelatin contraction may account for the discrepancy

Variation of Δ with Wave-length The values of Δ for a number of plates of various kinds were determined in the pre-

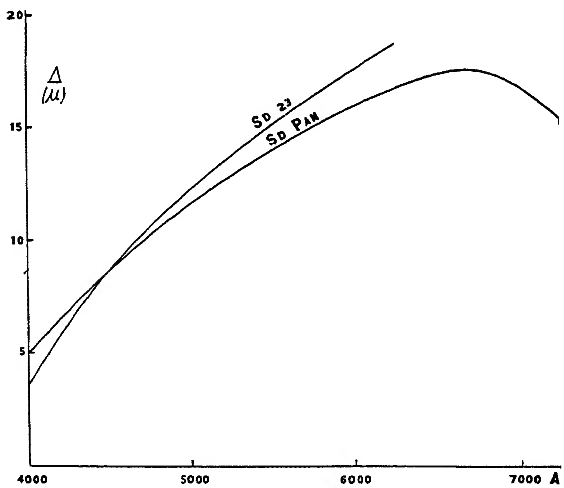


FIG 41
Variation of Δ with wave-length

cision camera, various color filters being used The curves thus obtained are shown in Fig 41 Δ is seen to be smallest in the violet, agreeing with theory

The extent to which the measured values of $\triangle$ depends on the optical system and its accuracy of focus has not yet been determined. In the case of printing *by contact with the emulsion of a specially prepared metallic slit*, more fully described in the next chapter, it can be taken for granted that the "optical" turbidity is negligible, and therefore the measured turbidity would be that of the emulsion itself. By comparing values thus obtained with those obtained by the use of telescopes, the relative importance of optical turbidity and emulsion turbidity may be judged. Many determinations of $\triangle$ for contact slit images were made for various wave-lengths on two emulsions, Seed 23 and Seed Panchromatic. Strictly monochromatic illumination was obtained by the use of a monochromatic illuminator. Development was in caustic hydroquinone. The mean results are shown in Fig. 41, smooth curves being drawn through the individual values, of which there were ninety-eight in the case of Seed Panchromatic. Comparing the values of $\triangle$ scaled from this diagram with values obtained in the precision camera, which are given on pages immediately preceding, it is seen that there is a substantial agreement. The conclusion is that for well-corrected telescopes at least, the optical turbidity is negligible, the actual turbidity obtained depending solely upon the physical characteristics of the emulsion.

Fig. 42 shows the intensity of the light distribution downward in an emulsion, computed from Bloch and Renwick's equation 46. W is taken as unity, which is a close approximation, and the thickness of the emulsion equal to 20μ. The distribution curve for Bouguer's law (A) is shown for comparison. Assuming each layer of grains equal to 2μ in depth, it is seen that the absorption in the first layer of grains at the surface is twice as much from Bloch and Renwick's equation as it is from Bouguer's. The same steep drop in transmission is shown by Bloch and Renwick's curves as is shown by the sideways transmission curve computed from the Greenwich equation 44. Reasoning from analogy, it is probable that in the first few layers of an emulsion, the transmission downward is more nearly in accord with Bouguer's than with the Bloch and Renwick's equation.

Image Growth Depending on Variation of Exposure Time. It is important to investigate the relation between the diameter of an image and the time of exposure, assuming the light intensity constant. In this case t must replace I as the variable in the equation,

$$i = I\ e^{-\kappa x}.$$

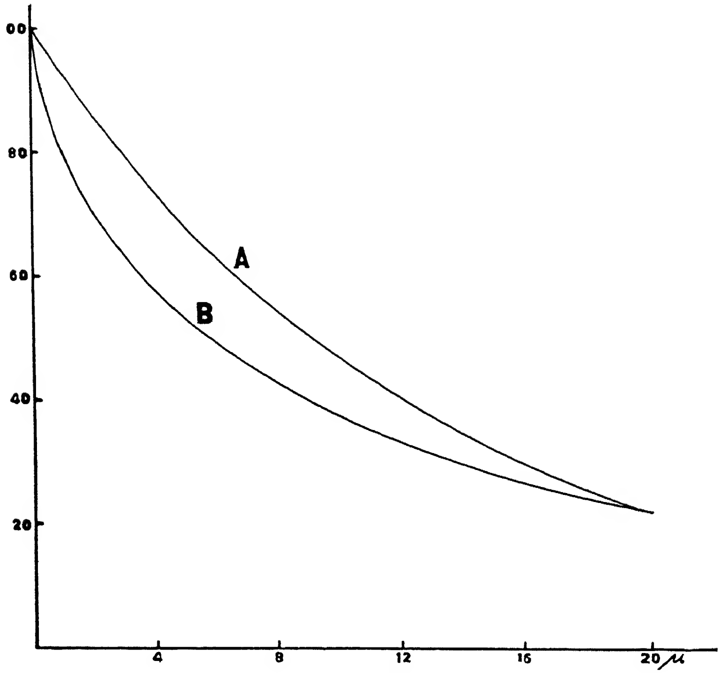


FIG 42
Light intensity in an emulsion

The relation which it is necessary to assume between I and t may be any of the three forms already presented, namely (1) reciprocity, $I\ t = C$, (2) Schwarzschild's form, $I\ t^p = C$, and (3) Kron's more complicated relation, given on p 63 For the present Schwarzschild's form will be taken, which includes the reciprocity form as a special case We have then, since $i\ t^p =$ constant,

$$I\ e^{-\kappa x}\ t^p = C',$$

or, taking logarithms and reducing,

$$x = C + \frac{p}{\kappa} \log t \tag{48}$$

If I and t vary simultaneously, the equation becomes

$$x = C + \frac{1}{\kappa} \log (I\ t^p) \tag{49}$$

An accurate method of determining p thus presents itself. If two series of star images are obtained by a logarithmic increase of the intensity and of the exposure time respectively, and the Scheiner equation (35) formed for each series, then p is the ratio of the coefficients of log t and log I. This corresponds to the densitometric method of obtaining p (p. 67) where it was shown that p was the ratio of the two gammas.

In practice the series of images is usually made by doubling the exposure time from one image to the following. Assuming, as we may, that Scheiner's law holds, let Δ = increase in the diameter of the star image, in microns, for each doubling of the exposure time, the average value being taken. This amounts to plotting the entire series of measures, drawing a straight line through the points, and measuring its slope. We derive at once:

$$\Gamma = \frac{\Delta}{\log 2},$$

$$\frac{p}{\kappa} = \frac{\Delta}{2} \frac{(Modulus)}{\log 2}, \tag{50}$$

from which, substituting numbers,

$$\kappa = \frac{1.386}{\Delta} p. \tag{51}$$

If the value of κ is desired when the base is 10, instead of e, as in the next chapter, corresponding to

$$I = I_0 \, 10^{-\kappa x},$$

the value of κ is

$$\kappa = \frac{0.602}{\Delta} p. \tag{51a}$$

In case the series of images is made by doubling the intensity, the same equations hold, with p equal to unity.

It is to be noticed that the law governing the increase in size of photographic images which takes place with increase in exposing intensity does not depend in any way upon the law of photographic action, while, on the other hand, when the increase in size is brought about by an increase in exposing time, there is a dependence, as shown.

It is quite probable that there are secondary development peculiarities such as those for example taking part in the

Eberhard effect and the Kostinsky effect which enter into the phenomenon of the growth of photographic images It is not at all unlikely that phenomena of this sort affect star images which have received long exposures, at least partly accounting for the modification of the rate of growth which is embodied in the Greenwich equation

We are now in a position to understand more clearly that branch of the subject of astronomical photometry depending upon the growth of images The sensitivity of the method is clearly governed by the magnitude of the coefficient Γ in equation (42) Since from equation (50), Γ or Δ is inversely proportional to κ it is necessary that κ be small This means that the image itself must not be sharp, since, for a given x, $e^{-\kappa x}$ is the nearer unity the smaller κ is, and accordingly I is greater For very sharp images, Δ is about 10μ, which is too small for accurate photometric work as will be shown For, as a simple calculation shows, an error of one micron in the measurement of the diameter, which is the probable error of such measurements even under the best conditions, corresponds to an error of 7 per cent in light-intensity and 0 075 in stellar magnitude Now this also is about the probable error in stellar magnitude due entirely to plate errors so in this case we are increasing an already large probable error to $0\ 075\sqrt{2} = 0\ 106$ magnitude Most photometric work of this character, however, is carried on with telescopes of long focal length in which, due to increased optical aberration of all kinds and to magnification of the unsteadiness of the air, the sharpness of stellar images is very much reduced The value of Δ in this case is in the neighborhood of 30μ Repeating for this case the calculations just made, assuming the same accuracy of measurement, the probable error becomes 0 025 magnitude The total probable error is then

$$\sqrt{\ 075 + \ 025} = \ 079 \text{ magn },$$

or, the plate error is increased by only an insignificant amount, so that the sensitivity of the method is ample

It will be of interest to deduce the sharpness curves for the two cases just considered, $\Delta = 10$ and $\Delta = 30$ From equation (51), assuming $p = 1$, we find

$$a = 0\ 1386, \quad a = 0\ 0462$$

The equations of light-intensity are then

$$\frac{I}{I_o} = e^{-0\ 1386x}, \quad \frac{I}{I_o} = e^{-0\ 0462x}$$

Another interesting case of the applicability of the method of determining the light distribution at the edge of an image is that of wave-length distribution, for it was found that Δ varied with the color or wave-length of the light. (See Fig. 41.) The light distribution curves, shown in Fig. 43, are plotted from the following data and the Scheiner equation:

λ	Δ
400$\mu\mu$	6μ
480	11
640	15

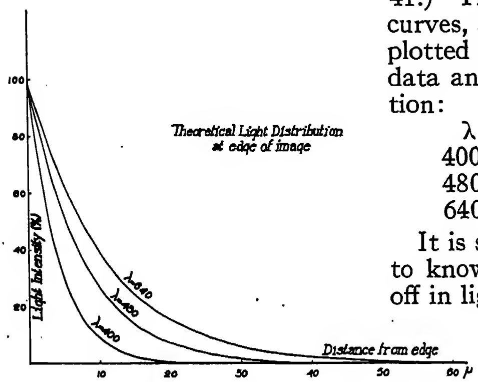


FIG. 43

Variation of light distribution curves with wave-length

It is sometimes of interest to know the rate of falling off in light intensity without plotting the curve. A simple rule is the following: At a distance Δ from the edge the light is in all cases 25 per cent of its value at the edge. The proof is as follows:

$$\Delta = \frac{2 \log_{10} 2}{\kappa \,.\, Mod.} = \frac{2 \log_e 2}{\kappa}.$$

The light intensity I, at a distance Δ, is given by:

$$I = I_0 \, e^{-\kappa\Delta} = I_0 \, e^{-2 \log_e 2};$$

taking logarithms,

$$\log_e \frac{I}{I_0} = -2 \log_e 2 = \log_e \frac{1}{4},$$

$$\text{or,} \qquad \frac{I}{I_0} = \frac{1}{4},$$

which proves the theorem.

It will be shown in the next chapter how the determination of Δ leads to a knowledge of the sharpness of the photographic image.

CHAPTER IV

Sharpness and Resolving Power

As soon as the use of the photographic dry plate had become established as a recording medium in scientific investigations, a knowledge of the physical properties of the various classes of plates became necessary In addition to numerical specification of the factors of speed, contrast value, and size of grains, a numerical specification of the resolving limit, or resolving power was required Defining resolving power qualitatively as the ability to show fine detail it is apparent that the resolving power of the photographic plate need be only as great as that of the optical system in connection with which the plate is to be used It was this consideration which guided the first investigator of the subject of the numerical specification of resolving power While undertaking a comprehensive survey of the optical properties of the modern spectroscope, F L O Wadsworth[1] derived a formula for photographic resolving power, based on some general considerations which are, however, difficult of acceptance He was lead to the result that the resolving limit is four times the diameter of the grain of the particular plate used, or, more specifically, parallel lines or small point images are separated when the distance between their centers is not less than four times the grain diameter Wadsworth took the grain diameter, presumably of a fast plate, to be 5 2μ, so that the resolving limit for such plates was 0 021 mm , which means that a line screen of 48 lines per millimeter is resolvable This is in fair agreement with results obtained by most present day workers in so far as the resolving power of fast plates is concerned, but does not agree with present day measures of grain diameter, which are but one-half the value quoted by Wadsworth

In deriving the above formula for resolving power, Wadsworth expressly stated that he avoided consideration of the effect of "photographic irradiation" To what extent this factor is operative has been considered by a number of investigators whose results and conclusions are now to be summarized in historical sequence

From a study of the grain and resolving power of the collodion plate, M Monpillard[2] was enabled to deduce the

[1] Wadsworth, F L O The modern spectroscope XVI and XVIII Astrophys J 3 188 328 1896

[2] Monpillard M , Experiments on the grain of silver images obtained in the wet collodion process Brit J Phot 54 936 1907

important truth that other factors besides grain-size must determine the resolving limit. He produced collodion plates with grain-size varying between the limits 0.4μ and 2.5μ and studied the corresponding resolving limit. While, in general, higher resolution was obtained on the plates of finer grain, there were some exceptions which showed the importance of other factors. In this connection the author remarks that it is well known that the grain-size of collodion plates may be considerably larger than that of fast dry plates and the resolution be very much better.

It appears to have been generally recognized that the spreading or increase in size of an image as the exposure is increased was responsible in a large measure for the resolving power of plates. It was known that this arose in two ways: (*a*) reflecting back (from the rear surface of film or plate) of the light which had filtered through the emulsion; (*b*) sideways scattering of the light in the film itself, due to the silver bromide grains reflecting and diffracting the light in which they are bathed. The effect of the reflected light in (*a*) can always be made negligible, so that it was important to investigate (*b*) which is the true turbidity or scattering effect.

The investigation of the turbidity properties of various emulsions was first undertaken by Mees[1] who, by an ingenious experimental device, made each emulsion write its own turbidity record. This was accomplished by photographing with a highly corrected lens, a fine slit, behind which had been placed a neutral wedge, so that one end of the slit transmitted only a small fraction of the light transmitted by the other end (one-sixtieth in this case). On account of the spreading of the light within the emulsion a "tadpole" developed image was formed. The exposure on each plate was so regulated that a barely perceptible image was produced at the point where the image of the dense end of the slit was formed on the plate. A number of these tadpole images are reproduced in Fig. 44.

From a study of these images, Mees concluded that spreading or turbidity does not depend on grain size alone. For example, he found that a chlorobromide plate having a diameter of grain of 0.8μ gave much more spreading than a process plate having a grain size ranging from 1.0μ to 1.5μ (see Fig. 44). He concluded that irradiation is composed of two factors: (1) reflection from the grains; (2) diffraction by the grains. In plates of large grain, reflection is the predom-

[1] Mees, C. E. K., On the resolving power of photographic plates, Proc. Roy. Soc. **83A**: 10. 1909.

inating factor in turbidity, while in fine grain plates the scattering is caused largely by diffraction With a diminution in grain size scattering by reflection decreases, while scattering by diffraction increases He concluded

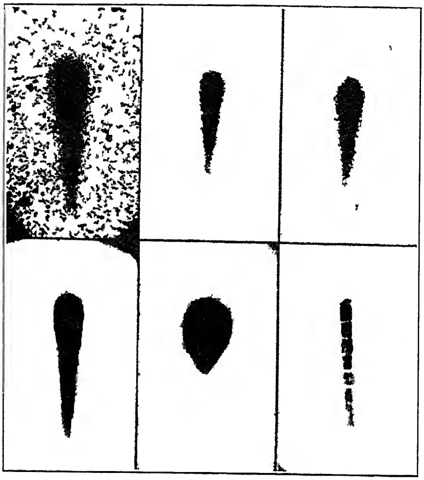
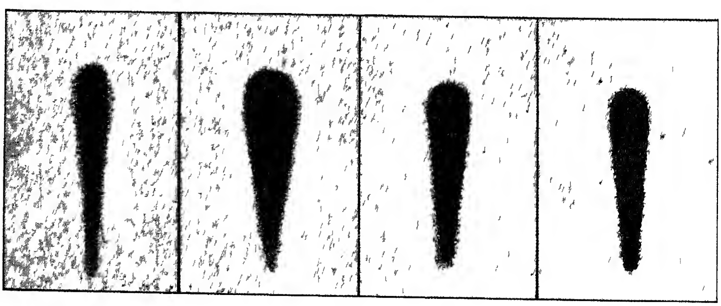


Fig 44
Tadpole method of showing turbidity

1 "Diffraction scatter will become less as the wave-length of the incident light increases Reflection scatter will be unchanged by an alteration of wave-length "

2 "Diffraction scatter will be small upon the surface of the film, and will grow as the film is penetrated Reflection scatter will be nearly constant throughout the film "

To verify these laws some experiments were made on red-sensitive plates In order to verify (1) exposures were made in succession through two screens, α, transmitting 6600A to 7200A, and Θ, transmitting 4000A to 4500A Results are shown in Fig 45, which reveal little if any difference in scatter on the process plate, but a marked difference on the fine-grain lantern plate, the scatter with red light being notably less To verify the surface phenomenon noted under (2) above, recourse was had to photographing in a photomicrographic



α Θ Fig 45 α Θ
Tadpole images in process and lantern plates

camera the tadpole image as it appeared on various kinds of plates. In this way the appearance in different *planes* in the emulsion could be studied. With the chlorobromide plate the very interesting result was obtained that a photograph of the surface showed little scattered light, while a photograph on the back of the emulsion showed an extensive scatter. This differentiation between the front and back was not shown on the coarser grained process plate. The differences between the two kinds of scatter, important from both a theoretical and a practical standpoint, thus appear to have been well established.

Passing to the more immediately important subject of resolving power, to which these introductory experiments and conclusions lead, Mees concluded that "the resolution of a photographic plate is dependent on the amount of irradiation displayed by that plate." Experimental determinations of resolving power numerically expressed were made by photographing with the same highly corrected lens reduced copies of one-half of a halftone screen. Defining resolving power as the number of lines per millimeter just separated on the plate, it was found that the resolving power of the average fast plate was 33, while that of the average fine-grain plate was about 55. Using red light, the resolving power was doubled. He remarks: "The resolution on the surface of a fine-grain plate will obviously be much greater than this as is shown by the very high resolving power possessed by the fine grained 'albumen' plates which are developed by the deposition of silver from an acid silver solution." However, the author was not able to prove that the physical development of lantern plates increased their resolving power, although no trace of grain was visible in a 2 mm. objective. Lastly, making use of the idea that a thin film gives high resolution, a plate was coated very thinly with fine-grain lantern emulsion, made panchromatic. The resolution of this plate for violet light was found to be 125; for red light it had the extraordinarily high value of 250.

Writing some years later on the same subject, discussing in particular the relative effect of turbidity and opacity on resolving power, Mees[1] states: "A typical example . . . is the wet collodion plate, in which the turbidity is considerable, but where the opacity of silver iodide for blue-violet light is so great that the resolving power is very high. In a Lippmann emulsion the resolving power is high if the emulsion is very

[1] Mees, C. E. K., The physics of the photographic process. J. Frankl. Inst. **179**: 141. 1915.

clear because the turbidity is very small, but the opacity is also small, and the slightest increase in turbidity may make the resolving power very low An interesting experiment which I have made was as follows I made a Lippmann emulsion, coated it, and found the resolving power, which was very high Some of the emulsion was slightly warmed so that it increased in speed somewhat and showed slight turbidity, and although the increase in speed was quite small, the resolving power was now very poor By further heating the emulsion so that it became visibly opaque, with a considerable increase of speed, although the turbidity was now high, the opacity had risen sufficiently to give the emulsion a considerably better resolving power than after the first heating had taken place "

Interesting observations on Mees' theory of reflection and diffraction scatter of light within a photographic emulsion have been made by Professor A W Porter[1] who remarks in this connection "The full elucidation of the behavior of plates in this respect would be a matter of very considerable interest, both theoretically and practically Unfortunately, the size of grain in a photographic plate is of such a magnitude as to introduce the greatest possible difficulty in working out a theory If the grains had been very large some success could probably be obtained by treating them as reflecting spheres For such spheres the amount of light diffused would be roughly proportionate to the total area of the grains On the other hand if their diameter were exceedingly small, so that many of them would be contained in a single wave-length of light, the question of the amount of scattered light is again a soluble one It has been discussed by Lord Rayleigh, and it turns out that for such small particles the amount of light scattered is proportional to the number of particles per unit volume, multiplied into the square of the volume of each, and that it is inversely proportional to the fourth power of the wave-length of light employed But it is not justifiable to assume that a blend between these two laws will hold in the case of the grains of intermediate sizes with which we are now concerned Moreover, a knowledge of the relative numbers of grains in the different plates is required before complete comparison can be made between them It can be shown that the breadth of the scatter will be small, both when this number is very small and when it is very large so that there is a certain intermediate number for which the breadth will be greatest, other things being the same When the number of grains is small, the

[1] Porter, A W The growth of the photographic image Brit J Phot 56 1010 1909

amount of scattered light is insignificant. On the other hand, when the number is great, although the amount scattered by any element is great, yet it can penetrate little distance because it is stopped by the grains in adjacent elements."

The various factors affecting the resolving power of photographic plates were given full treatment by Scheffer[1] who likewise realized the inadequacy of any treatment of the subject based on grain size alone, saying: "The size of the grain in the sensitive film cannot be employed without further qualification as a numerical measure of resolving power." He remarks that differences in color of various plates is a direct indication of differences in turbidity, one factor of importance in resolving power. "Fine-grain plates, in particular, allow light of short wave-length to pass much better than that of great wave-length. A similar effect comes in particularly in the case of plates bathed with sensitizers. . . . The turbidity of a medium is dependent on the size and number of the particles distributed in it. In a perfect solution the particles are so finely divided that the turbidity is nil. As the size of the particles increases the turbidity increases with them up to a certain value, beyond which further increase in the size of the particles diminishes the turbidity, ... assuming that the total mass of the substance suspended in a given volume remains constant. At a certain degree of turbidity the scatter of light in the film reaches a maximum. The turbidity can be easily recognized by observations on the film by transmitted light. . . . . The Lippmann film contains particles of extremely small size and shows a very slight turbidity. The limit of its resolving power certainly falls within the half wave-length of blue light. . . . . All my experiments have shown that the resolving power decreases as the turbidity increases. . . . The curve of gradation of the plate must be considered. The steeper it is, the sharper will be the edges of the image, and, therefore, the better the resolving power. . . . . The aperture of the cone of rays forming the image must be considered. The greater this angular aperture the more will the light be scattered in the film."

"Mention should also be made of the fact that the absorption of light by the film, especially in the case of dyed plates, is a notable factor. Light which is absorbed by the film will naturally be scattered far less in the plate and a better resolving power will be obtained in these circumstances. The best resolution will be obtained either with film which possesses

[1] Scheffer, W., On the resolving power of photographic plates. Brit. J. Phot. 57: 24. 1910.

the smallest degree of turbidity[1] In consequence of the small size of the light-sensitive particles, the scatter will be a minimum in this case and the developed grain will correspond very closely with the optical image formed by the light Next in order of excellence for resolving power come films into which the light does not penetrate, that is to say, where the light action is superficial, and neither down into the film nor sideways, or what practically almost amounts to the same thing, films in which the action of the developer is limited to the surface, that is to say, not penetrating into the lower layers of the film The resolving power of photographic films may then be stated to be dependent on the following (1) intensity, direction, and color of the light forming the image, (2) on the turbidity, gradation, color (absorption) and spectral sensitiveness of the film, on the size of the light-sensitive particles, and on the size and relative position of the developed grains to these latter, (3) on the manner of development, and (4) on the after-treatment"

With regard to the effect of development and of after-treatment which are included in the above summary, Scheffer states that development of the plate beyond a certain point causes the resolving power to decrease, and that intensification will improve resolution in the case of very fine grains, but if pursued beyond a certain point, will prove detrimental

Scheffer deduced important conclusions from photomicrographs of sections of films which have been subject to resolving power tests He states "It can be plainly seen that the resolved image lies only in the upper part of the film The finer the structure which is resolved, the more marked is this phenomenon In the case of specially fine structures on plates of high resolving power, the resolution can be completely destroyed by brief treatment with Farmer's reducer without greatly affecting the total density It should be specially noted that irradiation in the deeper portions of the film does not appreciably diminish the resolving power In correctly exposed and developed plates only the upper parts of the film show the black developed grains"

A number of experiments are described by Scheffer which illustrate various phases of the subject In one case he compared the resolving power of two emulsions in one of which the silver bromide grain was much finer than in the other Development was such that both plates developed to the same size of black silver grain The resolving power of the finer grained emulsion was found to be much higher of the two, proving

[1] Sentence does not appear to be complete—*Author*

that resolving power is to an extent governed by the size of the silver bromide grain and not by that of the developed silver grain

Concerning the effect of the color of the light on resolving power, Scheffer states "The resolving power in red light of the sensitized plates is the same as that in blue light of the non-sensitized plate This experiment proves that the said phenomenon is not simply caused by the fact that light of short wave-length is more scattered by diffraction than light of long wave-length " In this connection he finds the curious fact that bathing a plate in pinacyanol and thus making it red-sensitive causes a marked diminution in its resolving power under blue light illumination "Further experiments with sensitized plates of coarser grain have shown that under certain conditions light of short wave-length can give a better resolving power than that of long wave-length "

Reverting to a different phase of the subject, Scheffer finds that in determination of resolving power by means of gratings, the resolving power is not affected by relative alterations in the widths of the light and dark spaces but depends only on the period, or number of light or dark spaces per millimeter

The next important investigation of the subject is by E Goldberg[1] who restricted himself largely to the study of but one phase of the subject, the turbidity or spreading of the developed image, without consideration, except in a general way, of the separate factors producing the observed turbidity effects' In order to measure turbidity, Goldberg exposed plates behind a carefully prepared round hole in a metal plate, pressed against the emulsion By making a series of exposures on each plate with exposure time increasing logarithmically a series of images is obtained whose diameters increase regularly and can be measured with great accuracy The rate of increase in the diameter is taken as a measure of turbidity Exposures ranged for each plate from the threshold value or just enough to produce a discernible image, up to one hundred thousand times the threshold value For all of the kinds of plates, no spreading whatever could be discerned until the exposure reached a value ten times threshold At this point spreading began at a rate which was characteristic of the particular emulsion The grainless Lippmann plates were an exception, no perceptible increase in size being shown on the most prolonged exposure The results for various emulsions obtained by Goldberg are shown in Table 11 *I* is the

[1] Goldberg, E, On the resolving power of the photographic plate Phot J **36** 300 1912

increase in diameter in microns between the threshold exposure and 100,000 times threshold The writer has added, for its astronomical application, the last column, which is the increase in diameter per unit increase in stellar magnitude, computed from the numbers in the preceding column on the assumption that no increase takes place in the diameter until the exposure has reached ten times threshold, as previously noted by Goldberg, so that I really corresponds to 10,000 times threshold, or ten stellar magnitudes

TABLE 11

PLATE	I (μ)	$\frac{dI}{dM}$
Lippmann	0	0 0
Transparency	30	3 0
Process	44	4 4
Sigma Instantaneous	56	5 6
Portrait	74	7 4
Double Coated	97	9 7

Contrary to results obtained by many investigators, Goldberg finds that $\frac{dI}{dM}$ increases with the exposure, but uses it in subsequent discussion as though it were a constant, which he calls δ, the turbidity factor It has been previously shown by Scheiner[1] that δ represents the light distribution at the edge of an image, such as that of a star, on the photographic plate Defining the sharpness S of an image as the rate of fall of density outward from the image edge, Goldberg notes the following simple formula for its calculation

$$S = \frac{dD}{dx} = \frac{dD}{d \log I} \frac{d \log I}{dx} = \frac{\gamma}{\delta},$$

in which γ (gamma) is the contrast of the emulsion, which Goldberg calls the development factor From this he deduces the following important law "The sharpness at the edge of a point, a line, or a surface, is at its greatest with a given plate which possesses the smallest turbidity factor and the greatest development factor "

Goldberg recognized (l c) that this formula and rule does not apply to very light exposures, in which he finds the turbidity factor identical for all kinds of plates This, however, is the case of most interest, for the reason that sharp images are obtained in practice by light exposures, the sharpness varying,

[1] Scheiner J Die Photographie der Gestirne p 218

as is known, according to the plate used. He accordingly forsakes the conception of turbidity as furnishing an explanation of the phenomenon of sharpness.

Passing to the subject of resolving power proper, Goldberg says: "If we now indicate by the term 'resolving limit' the smallest diameter of a disk or the smallest width of a printed line which can be made, this resolving limit must naturally correspond with the size of the grain which is peculiar to the plate under trial. A resolving limit in the sense of Mees and of Scheffer, who by the term understand the smallest width of a photographed line just representable by the plate, would, therefore, not exist at all, or rather would be identical with the size of the grain." Discarding lens systems as useless in investigating resolving limit, he obtains photographic deposits by contact printing, using as a negative a silvered mirror in which fine scratches have been made by emery. He continues: "The first experiments gave the surprising result that the resolving limit is different with different plates, and that it is quite unimaginable with many plates to obtain lines in the print which correspond in their width to the size of grain. The conception of Wadsworth is therefore totally wrong." The failure is most pronounced in high speed emulsions. Goldberg explains this result as being due to the non-homogeneity of the emulsion, which in the case of fast plates is a mixture of sensitive and insensitive grains. Accordingly, a highly sensitive grain lying just without the geometrical edge of the image will be affected sufficiently by the feebly scattered illumination to which it must be subject to make it developable. This, combined with groups of relatively insensitive grains lying just within the image edge, serves to explain the phenomena observed. Goldberg concludes that there is no relation between the resolving limit and the turbidity factor.

E. Lehmann[1] gives important data on the resolving power of collodion plates. He finds that the emulsion thickness is 3μ, the diameter of grain variant from 1μ to 3μ and the resolving power 132. Comparing photomechanical plates with collodion he finds the resolving power of the photomechanical the better of the two. This appears to be due to its smaller grain which is only one-half that of the collodion plate. On the other hand he finds that the sharpness of edges on the collodion plate is higher, which is an interesting case of the distinction between resolving power and sharpness. Some data

[1] Lehmann, E., On the resolving power of photographic films and the reproduction of fine detail. Brit. J. Phot. **60**: 7, 23. 1913.

on grain-size and resolving power of the albumen or Taupenot plates were obtained According to Eder[1] the grain on this plate, with alkaline development, is so fine as to be immeasurable Lehmann finds that the size of grain depends on the manner in which the plate is made and gives 0 2μ as an approximation to the grain-size The resolving power he finds to be too high to be measured with a lens, but states that it must be at least equal to 600, since a Rowland grating of 12,500 lines per inch has been successfully printed on an albumen plate by contact printing

P G Nutting[2] has a short contribution to the subject of resolving power, which he conceives to be linked with density gradient or sharpness of image He states in this connection "Resolving power may be deduced at once from density gradient It is a given length (1 mm) divided by four times the distance within which density falls to a given specified percentage (say 98 per cent) of its normal value In the case of abnormally coarse grains and great underexposure, size of grain enters as a factor " Sharpness is computed, as Goldberg has already shown, by the equation

$$S = \gamma/\delta$$

Nutting states that in the case of normal or underexposures, the δ determined from the rate of growth of images is not applicable and that δ can only be determined in such cases by measuring S and γ To obtain δ directly is as yet an unsolved problem He measured the density gradients on several emulsions The edges measured were obtained by contact printing, a carefully prepared knife edge being pressed against the emulsion and exposed to parallel light He concludes from these measurements

"Up to a density of about unity there is no diffusion of the image properly speaking, only a raggedness due to the random distribution of grains of finite size" For densities between one and two the density gradient decreases, becoming constant for higher values Expressing the density gradient numerically, and in terms of grain-size, he finds that the maximum gradient (density unity) is 0 5 per unit diameter grain-size (1μ), or,

$$S = \frac{0\ 5}{d} \quad (d = \text{diameter of grain}),$$

exceptions being emulsions which are easily fogged, in which cases the gradient was found to be only one-tenth the above

[1] Eder s Handbuch Vol II p 52

[2] Nutting P G Photographic resolving power Phot J 38 265 1914

value He draws the useful inference that emulsions whose H and D curves show a long "toe" (p 127) give low gradients

The next investigation of importance is by Tugman[1] who conceives that resolving power is best investigated in an indirect manner through measurements of sharpness Concerning the work of his predecessors he states "The resolving power of photographic plates has hitherto been investigated by methods which have been suggested by general definitions " Following Nutting, Tugman makes use of a well sharpened straight edge, shown in cross section in Fig 46, contact prints being made by laying the straight edge on the photographic plate and exposing to a parallel beam of light The sharpness of the edges of the negative images obtained in this way was measured in two ways (*a*) By projecting a highly magnified image of the edge upon the nose of a Martens photometer, over the aperture of which narrow slits of equal width (0 5 mm) were placed The photometer is set in a traveling carriage attached to a screw with a divided head, so that it can be moved across the image of the edge which is to be measured, in a known manner The density at a number of points in the edge, which lie on a line perpendicular to it, is thus measured with respect to clear gelatin adjacent to the edge By plotting the readings and multiplying by the magnification, the rate of fall of density, or the sharpness, is thus measurable (*b*) The second method involved the use of the Koch registering microphotometer, an instrument specially designed and apparently well adapted to work of this kind In using the Koch instrument numerous difficulties were met with which are described by the author and which are not necessary to enter into here Method (*a*) was finally found to be preferable An objection to both methods was the difficulty or impossibility of obtaining any readings for even moderately high densities, so that only the lower portion of the sharpness curve could be obtained The trouble was caused largely by scattered light, which in a high power projection system like the one employed was considerable Only densities of about 1 2, corresponding to an opacity of 16, could be read For the same reason the sensitivity for densities from 1 0 to 1 5 was low, so that sufficient accuracy was not obtainable

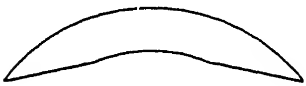


FIG 46
Cross section of knife

[1] Tugman, O The resolving power of photographic plates Astrophys J **42** 331 1915 An adaptation of the Koch registering microphotometer to the measurement of the sharpness of photographic images Ibid p 321

A large number of curves obtained in this way are given by Tugman, which are of interest and importance as being the first attempts in the direction of actually depicting sharpness curves From these curves, several conclusions of importance are drawn

It was found that the time of development does not affect the sharpness, defining sharpness as the slope of the steepest portion of the density gradient curve That the sharpness for long development is no greater than for short is explained as due to the increase in thickness of the developed layer with increase in development time Concerning observed variation of sharpness with wave-length the author remarks, "The variation of density gradient with wave-length of exposing light may be explained by the difference in optical opacity of the emulsion for various wave-lengths The variation of photographic gradient is not enough and not in the right direction to account for the variation of resolving power with wave-length In general the sensitiveness curve of a plate exposed to green light is steeper than the curve of the same emulsion exposed to violet light Also the depth of penetration of green light is greater than that of light of shorter wave-length Therefore, if the optical opacity for the long waves is less than for the short waves, the light gradient in the emulsion film will be less steep for green light than for violet light, and, consequently, there will be more spreading of the image made by green light The larger value of the photographic gradient is overbalanced by the lower value of the light gradient This explanation assumes that the scattering of light by the silver halide grain does not vary appreciably with wave-length This assumption is justified by the fact that it is the short waves which usually are scattered most by heterogeneous media and, on this account, the resolving power of a photographic plate should be greater for long waves than for short waves Further evidence that the opacity of the emulsion is a more important factor in the light gradient than diffusion is given by the curves of a process plate immersed in yellow dye and exposed to violet light In this case the diffusion could not be changed, but the opacity was increased with a corresponding increase in resolving power "

From photomicrographs of sections of edges the author finds that the greatest spreading occurs at the top of the film "This distribution suggests that the intensity of light in the emulsion at any point is given by the equation

$$I = I_0 e^{-K(x+y)}, \tag{52}$$

where x is the distance down through the emulsion and y is the distance out from the image and measured from the region of uniform exposure." (See p. 41).

Kenneth Huse[1] attacks the problems of resolving power in a direct manner and obtains a mass of useful data, a summary of which will be given. His results are based on measurements with the fan test-object, devised and used by Mees for the determination of resolving power. In conjunction with a suitable illumination system the test-object was placed at one end of a cast iron tube 6 meters in length, at the other end of which was placed a Fuess single-achromat of 6-inch focus, of aperture ratio $f/5.3$ and focal length 15.3 cm. Accurate focusing was obtained by a slow-motion screw. The reduction was 40 diameters. The fan test-object is shown in Fig. 47. The resolving powers of a variety of emulsions of different

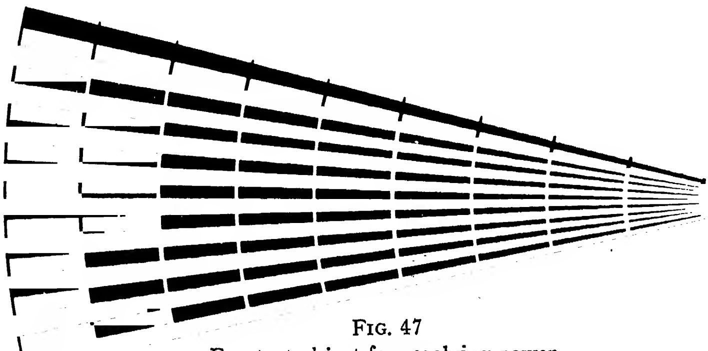


FIG. 47
Fan test object for resolving power

types are tabulated below. Development was in pyro, with the exception of the albumen plate, which was developed physically. White light exposure was presumably used. The resolving power of the lens was about 400, which is well above the resolutions to be measured. The following results were obtained:

TABLE 12

Plate	H. and D. Speed	Resolving Power
Albumen	0.01	125
W. and W. Resolution	3.0	81
W. and W. Slow Process Pan	5.0	67
Seed Lantern (Yellow Label)	6.0	62
Positive Motion Picture Film	10.0	42
Seed 23	150.0	35
W. and W. Panchromatic	200.0	31
Seed 30	400.0	29
Seed Graflex	450.0	25

[1] Huse, K., Photographic resolving power. J. Opt. Soc. Amer. 1: 119. 1917.

A study was made of the variation in resolving power with exposure and with development time The relations obtained are shown in Fig 48 Very short development time is seen to

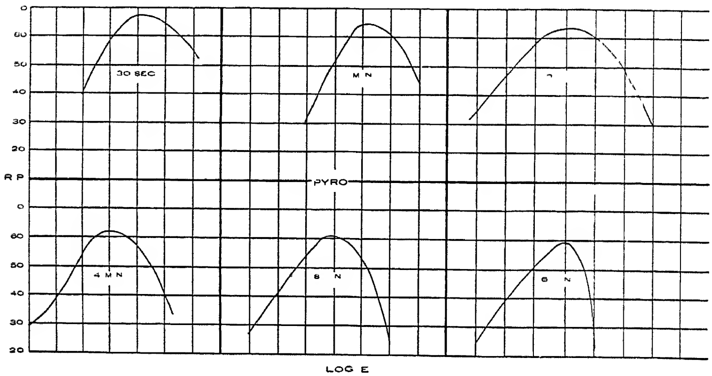


FIG 48
Resolving power depending on exposure and development

give the best resolution The effect of development time was investigated for several other developers The results are exhibited graphically in Fig 49 Thus the resolving power

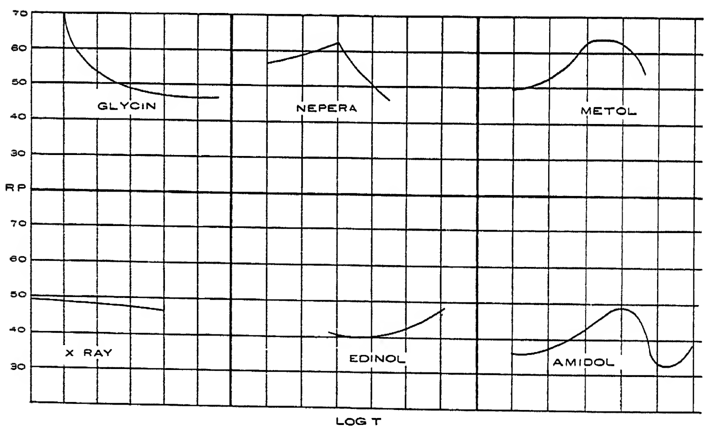


FIG 49
Resolving power depending on development time

characteristics of different developers, from the figure, are seen to vary widely The author explains the secondary rise

in resolving power in the case of amidol developer as being due to a diminution of fog with increase of development.[1]

To determine dependence upon developer the resolving powers of twenty developers were determined under the best conditions peculiar to each. The results are given in Table 13.

TABLE 13

Developer	Maximum Resolving Power	Exposure (in sec.)	Development (in min.)
Pyro caustic	77.0	4	2
Glycin	69.0	3	1
Hydroquinone	64.0	3	2
Pyro	64.0	3	2
MQ 25	64.0	3	2
Metol	63.0	3	2
Nepera	62.0	3	2
Pyrocatechin	62.0	8	2
Pyro metol	62.0	8	2
Eikonogen-hydroquinone	61.0	4	3
Ferrous oxalate	61.0	2	4
Caustic hydroquinone	57.0	4	2
Eikonogen	57.0	4	4
Kachin	54.0	2	5
Amidol	51.0	2	4
Process hydroquinone	50.0	2	2
Ortol	49.0	4	2
Rodinal	49.0	8	2
X-ray powders	49.0	3	1
Edinol	47.0	4	16

The author remarks, "From this table (Table 13) it is seen that resolving power is not an inherent property of the plate or film but is very dependent on the reducing agent employed in development. Thus, on the material used (Seed Lantern) in these experiments, resolving power values were obtained varying from 47 to 77 by simply altering the developer. . . . . The values of exposure and time of development tabulated above necessarily apply only to the particular conditions of these experiments, serving merely to show the dependence of good resolution on these factors as well as their variation with the reducer or developer used. An attempt was made to evolve some relation between this ability of a developer to give good resolution and its reduction potential, but none was obtained."

Lastly, experiments were made to find a relation between resolving power and the wave-length of the exposing light.

[1] This phenomenon is best seen in the bright ring often surrounding dense and fully developed images. It is known as the Mackie line.

Three monochromatic filters were employed having maximum transmissions respectively at 4550, 5300 and 7000A. The fan test-object and illuminating apparatus described above were used. Photographic tests, however, showed a distinct gain in resolving power by stopping down from $f/5.3$ to $f/8$. To give some idea of the delicacy of adjustment necessary in work of this kind, it was found that working at $f/8$ an error of focus of 0.03 mm can be detected by its effect on the resolving power. The camera, therefore, must be focused separately for each color. The color-curve of the objective thus found agreed exactly with that found on a precision lens bench. It was concluded from these experiments that resolution is best in the blue, falls off in the green but recovers in the red. This phenomenon can be explained by referring to the curve of wave-length and turbidity, and the curve of wave-length and γ (Figs. 58 and 59). In the violet the turbidity factor is high and γ low, while in the red the turbidity is low and γ high. Combination of the two curves gives a minimum in the green, agreeing with observation. Fig. 50 shows the observed variation of resolving power with wave-length for several classes of emulsions found by the present writer.

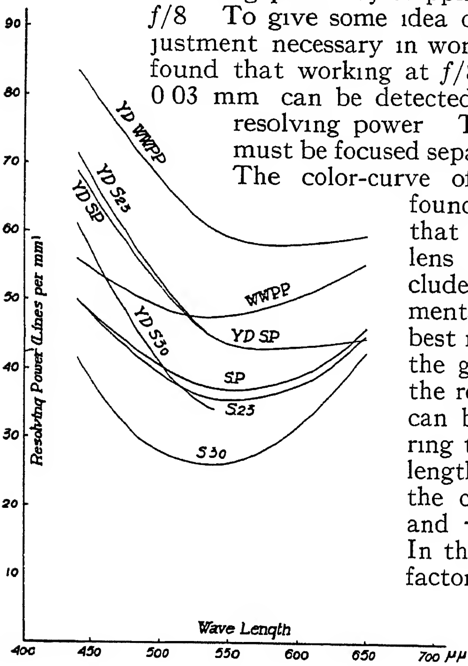


FIG. 50

Resolving power depending on wave-length

SHARPNESS OF PHOTOGRAPHIC IMAGES

Sharpness is primarily a sensation, which, however, can be correlated with physical quantities and thus expressed numerically. Consider for simplicity a photographic image having a straight boundary. Its sharpness clearly is the rate of change of the visual sensation of its brightness in a direction perpendicular to its edge. But, within a considerable range, the visual sensation of brightness is proportional to photographic density. Accordingly sharpness is definable by the equation

$$S = -\frac{d\,D}{d\,x}; \tag{53}$$

since and

$$\left.\begin{aligned} D &= f\,(I, t, a, b, \ldots); \\ I &= \phi\,(x), \end{aligned}\right\} \tag{53a}$$

from which

$$\frac{d\,D}{d\,x} = -\frac{d\,D}{d\,I}\;\frac{d\,I}{d\,x},$$

or

$$\frac{d\,D}{d\,x} = -\frac{d\,D}{d\log I}\cdot\frac{d\log I}{d\,x}. \tag{53b}$$

This relation was first pointed out by Goldberg.[1] The assumptions made in deriving it must not be overlooked. It is tacitly assumed that the parameters a, b . . . are *not* functions of x, the distance from the image to the point considered. In general this is not true, for there are development factors which are doubtless functions of the distance from the edge of the image. For example, there is the Eberhard effect, whose action in this case is a function of the sharpness itself. These are, however, secondary effects with which we are not now concerned. It is well, nevertheless, to point out the limitations of equation (53*b*), which have not heretofore been recognized.

The Sharpness Curve. The complete sharpness curve can be obtained by combining the two equations (53*a*) into one. Several forms of the f function have been given in the preceding chapter. The Hurter and Driffield form on account of its convenience (p. 52) will be utilized. The form of the ϕ function has also been obtained (p. 93). It is possible to derive a very simple rule for obtaining the sharpness curve, without reference to any theoretical expression for the f function. The "characteristic curve" of an emulsion, or the H. and D. curve is a graph of density as ordinate and log It as abscissa. Or, if t is constant,

$$D = \Psi\,(\log I); \tag{54}$$

since (p. 93)

$$I = I_0\,e^{-\kappa x},$$

$$\log I = \log I_0 - \kappa x;$$

whence

$$D = \Psi\,(\log I_0 - \kappa x),$$

which shows that the density is the same function of $\log I_0 - \kappa x$ as it is of $\log I$. Since the sharpness curve is density plotted against x, it is seen at once that in order to obtain this curve

[1] Goldberg, E., l. c.

it is only necessary to expand the characteristic curve in the direction of the X axis by the factor κ, or more strictly, the new horizontal scale is κ times the old one Since κ is less than unity, generally a small fraction, this means a compression of the characteristic curve Suppose an image is exposed and developed to a density D which is less than its maximum density Mark the point D on the characteristic curve The sharpness curve then is the entire portion of the H and D curve which lies below D, compressed by the factor κ (It is to be noted that if common logarithms are used in drawing the characteristic curve, which is customary, κ must be replaced by $\kappa Modulus = 0\,434\kappa$) If D is moderately large, the straight line portion of the curve will be included, in which the density gradient is a maximum, being in fact equal to gamma In this case, maximum sharpness will be obtained, the value being given by

$$S_{max} = -\gamma\kappa \qquad (55)$$

In Fig 51 is shown a series of sharpness curves, which were computed from the Hurter and Driffield equation (p 52) It is necessary to give numerical values to the three parameters in this equation O is put equal to 100, $\gamma = 1\,65$, from which $p = 1\,90$ Defining $Q = q\,It$, the value Q_0 of Q for the lightest exposure was chosen $Q_0 = 0\,001$ The values for the suc-

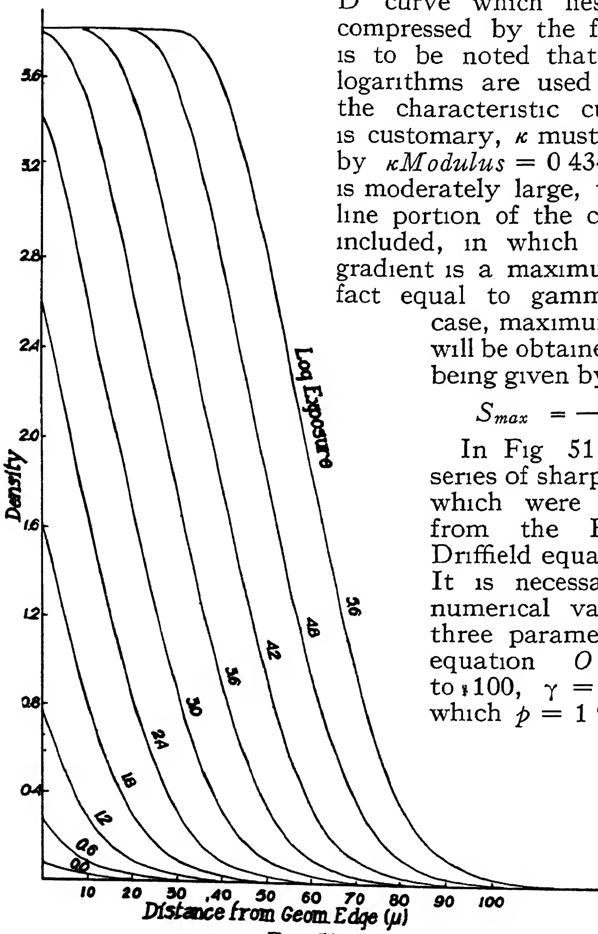


Fig 51
Theoretical sharpness curves

ceeding curves were $4Q_0$, $16Q_0$, $64Q_0$, ... $264144Q_0$. The value of the light distribution parameter κ, reduced to common logarithm was chosen equal to 0.058 ($\Delta = 10.4\mu$) p. 105. It is seen from the figure that for light exposures the density-gradient or sharpness is low, and that even for the useful density of unity, the sharpness is only 70 per cent of the maximum. The formula for sharpness which is usually given is therefore true only for high densities, if the Hurter and Driffield equation is correct, and is seen to be considerably in error in the useful cases of moderate densities of the image, which are the rule in cases of high resolving power. But in the case of most emulsions the slope of the characteristic curve is greater for low exposure than the H. and D. equation indicates. This means that the formula for sharpness $S = -\gamma\kappa$ is correct over a wider range of exposures than the formula gives, which is borne out by measurements of sharpness.

The foregoing developments indicate the type of emulsion which should be used to secure sharp images. First, it must be one of high γ and low turbidity, turbidity being defined as the reciprocal of κ. Secondly, it should belong to type A, Fig. 52, having a minimum amount of "toe." For in cases of

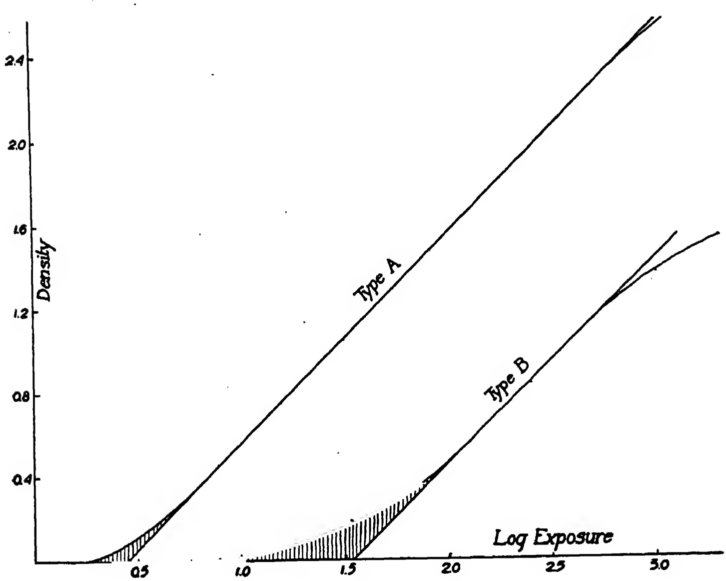


FIG. 52
Emulsion types, characteristic curves

high resolution that arise in practice the density of images is low, generally less than unity, so that it is very important that a maximum slope of the characteristic curve should be reached at a comparatively low density For this reason emulsions of type B are quite unsuited to give sharp images

Since, especially in the case of emulsions of type B, the density gradient $\frac{dD}{dx}$ varies considerably on passing out from the true edge of the image, there is a question as to what value of $\frac{dD}{dx}$ to take for the sharpness Must the maximum value be taken? This cannot be answered *a priori*, without experimental data Cases of this kind are met with in practice, a very pronounced toe appearing in the measured sharpness curve under certain conditions

Measurement of Sharpness So far as the writer is aware the only measurements of sharpness of image so far made are those by Tugman and Nutting In collaboration with Nutting, the writer developed and perfected Tugman's second method Largely because of scattered light, the methods used by Tugman were incapable of measuring densities greater than about 1 2, all higher densities registering the same value This lack of sensitivity is a serious drawback to accurate measurement of density-gradient The improvements designed were in the direction of increasing the sensitiveness Success was attained to such a degree that it was possible to measure with the perfected set-up densities up to about 3 0 This was attained by using an arc as the source of illumination, by replacing the high-power microscope-objective having many air-glass surfaces by one of lower power with few free surfaces, by removing the eyepiece of the microscope, obtaining magnification by an increased "throw", and lastly by compressing the magnified image in the direction of its length by means of a cylindrical lens On account of the effects of residual scattered light, and the fact that illumination was by specular light, it was necessary to calibrate the instrument to read diffuse densities[1] This was done by comparing a series of densities measured on a Martens photometer, used in the ordinary way, using completely diffused illumination, with the same series measured on the microphotometer Some details will now be given

[1] There is a difference in measures of density depending upon the character of the illumination whether by specular by diffuse light or by a combination of the two The ratio of specular to diffuse density which is greater than unity reaches in some cases a value of 1 5 It is generally called q

A photograph of the microphotometer, omitting condenser, is shown in Fig. 53. By means of the screw and divided head, the Martens photometer is moved across the magnified image

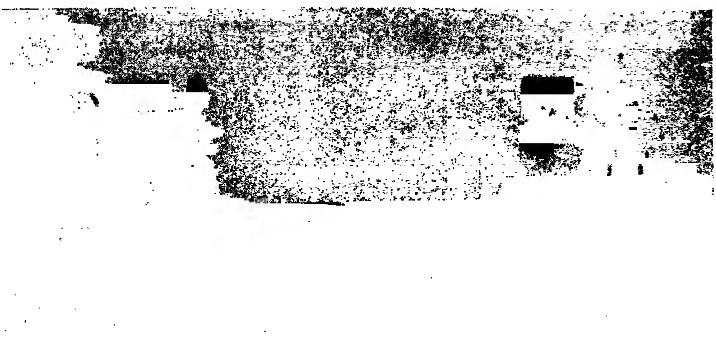


FIG. 53
Microphotometer

of the edge to be measured. In contact with the openings of the photometer is placed a light diffusing medium upon which the image must be focused. Tracing paper has been found best for this purpose as it gives sufficient diffusion without unduly reducing the intensity. A metal cap containing the slits fits over the end. These slits are of exactly equal width (0.57 mm.) and 11 mm. apart. On the outside of this cap is pasted a piece of paper acting as a screen which does not interfere with the slits. Ruled across its center is a line parallel to the slits. A sharp image of the edge to be measured is thrown on this paper, the edge being made to fall midway between the two slits with which it is placed parallel, through the intermediary of the line ruled on the paper screen. *T* is a blackened tube keeping out stray light. *C* is a cylindrical lens which is diaphragmed by an accurately cut rectangular mask 15 mm. wide. After the image has been properly focused, the tube *T* and cylindrical lens are placed in position. The cylindrical lens is not placed in the position of best focus, but is so adjusted that the light is concentrated in a diffused band about 2 mm. wide, this width being found to produce the best results. It is necessary to adjust the axis of the cylindrical lens. This is best done by substituting a razor blade for the edge being measured, which gives an image of such sharpness that it can be accurately adjusted parallel to the slits after the

cylindrical lens has been inserted Stops on the base keep the cylindrical lens in correct position

The microscope used as a projector is a standard type, with double-motion stage The eye end is removed The microscope is placed about five feet in front of the photometer, the distance varying with the magnification desired An arc of the automatic feed type, with a condenser, is used as the source of illumination A circular hole or aperture plate is placed in front of the condensing lens nearly in contact with it This diaphragm is focused by the substage condenser upon the focal plane of the microscope This is accomplished by first focusing the edge to be measured upon the photometer or upon a screen placed in front of the photometer, and then adjusting the substage condenser of the microscope until the borders of the diaphragm are sharply outlined on the screen along with the image itself Instead of using a condenser diaphragm a pinhole can be placed on the microscope stand just under the plate to be measured The object of diaphragm or pinhole is to reduce to a minimum the size of the illuminated area in the focus of the microscope objective As already stated, the principal difficulty to be overcome is that of scattered and reflected light The surfaces of the objective reflect back upon the plate which is being measured a large amount of light, thus enormously diminishing the real contrasts which are present in the plate This secondary plate illumination is plainly visible The larger the illuminated area of the image, the greater will be the amount of light reflected back and the greater will be the degradation of the contrast Similar actions take place within the microscope at each glass-air surface. The importance of reducing the number of these surfaces is thus seen For this reason the eyepiece of the microscope has been dispensed with entirely and a simple type medium power 16 mm objective, being relatively free from air surfaces, is employed Great care must be taken to keep all surfaces scrupulously clean

On account of the high magnification, an intense light source is necessary The brightness of the photometer field even with arc illumination is exceedingly low when densities above 2 are being measured One of the advantages of the cylindrical lens is increasing the field brightness, so that it is possible to read higher densities The contrast discrimination of the eye is low at low intensities so that any increase in brightness is of advantage A further advantage of the cylindrical lens is its integrating effect A section 15 mm long of the magnified image is compressed into a space sufficiently small to enter the

photometer field. Thus an average value of the sharpness is secured which is highly desirable.

Systematic Errors in Sharpness Measurements. A systematic error in measuring sharpness with this instrument has been found, depending on the magnification used. Results obtained from magnifications of 65, 110, 150 and 208 have been compared. The measured sharpness has been found to increase with the magnification. The increase found on passing from magnification 110 to 208, however, is so small that the error in using magnifications of 150 and 208 is negligible. It is found, for instance, that the measured sharpness at 150 is 30 per cent larger than at 65. Consequently, it is necessary to make the measures at higher magnifications. This applies only to the case of very sharp images, such as have been discussed so far. In the case of diffuse edges a lower magnification can be employed. Here the magnification error will probably not be present. The source of this error is not, at the present time, understood. The correction for slit-width is small, so that the error can not be due to this. In general, the slit-width correction is of sensible size only on the toe and shoulder of the density-gradient curve, where the variation of density is a maximum. In the case of sharp images of low density the correction to the density gradient is considerable, for the width of slit used. It is best applied by graphical methods.

To illustrate the precision to be expected of the method of measuring edge sharpness, an example was chosen of five edges which were printed on a single plate, the exposures ranging from 1 to 16 seconds. The plate was Seed 23, exposed to 4400 A, with development in pyro to a gamma of unity. The density readings are plotted in Fig. 54, the dots representing separate

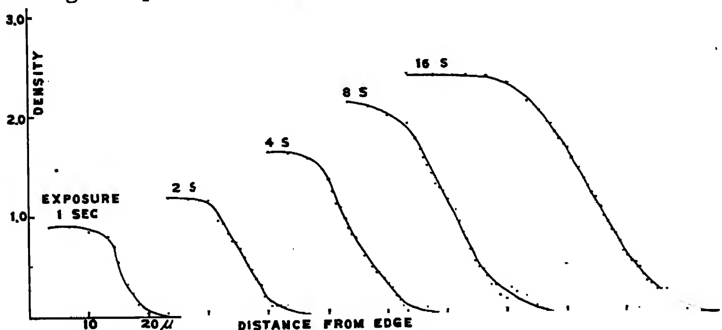


FIG. 54
Measurement of sharpness

readings The magnification used in this case was 150 Readings were taken at every tenth of a millimeter The precision of the curves is seen to be very high

Dependence of Sharpness on Exposure Time It is of importance to determine if there is any difference in sharpness between an image of low and one of high density In the case of images having a density less than about 0 8 it is difficult to secure exact measures, so that images of lower density than this will be excluded Comparison was made between images of density unity with images of density ranging from 2 to 3 As may be seen from Table 14, a variety of conditions was chosen, the plates being exposed at various wave-lengths and developed for variable times, the exposure times being in all cases adjusted to give nearly the same densities The plates used were Seed panchromatic Development was in caustic hydroquinone developer Each density gradient given is the mean of measurement of 10 edges The numerical values are differences in diffuse density between points 0 001 mm apart on the image itself, and are accordingly the values of sharpness of image

TABLE 14

DENSITY GRADIENTS

	$\lambda = 420\mu\mu$			$520\mu\mu$			$660\mu\mu$		
Development Time	¾ m	1½ m	3 m	¾ m	1½ m	3 m	¾ m	1½ m	3 m
Density gradient for Light exposure	107	136	143	051	059	069	053	070	082
Density gradient for heavy exposure	112	133	151	045	061	063	056	064	086

While the development and wave-length effects are seen to be present, no difference between the light and heavy exposure is indicated

The first piece of work attempted with the improved microphotometer was the determination of the sharpness of images for different developers and different development times, the latter corresponding to variations in gamma If the simple theory developed for sharpness is correct all developers should give the same sharpness provided development is carried to the same gamma The exposures were sufficiently long in all cases to produce high densities, but not long enough to produce halation An average of eight slit images were printed on each plate in the usual way, so that each density gradient is the mean of eight measured values Exposures were to monochromatic light of wave-length 4400A The increase in the

width of the slit for each doubling of exposure time was found to be $\Delta = 8.1\mu$. Accordingly the theoretical value of the sharpness is in this case (equations 51*a* and 55):

$$S = \frac{0.602}{\Delta}\gamma = 0.743\gamma. \qquad (55a)$$

This value of the theoretical sharpness is plotted in Fig. 55 along with the measured curves. The theoretical sharpness is

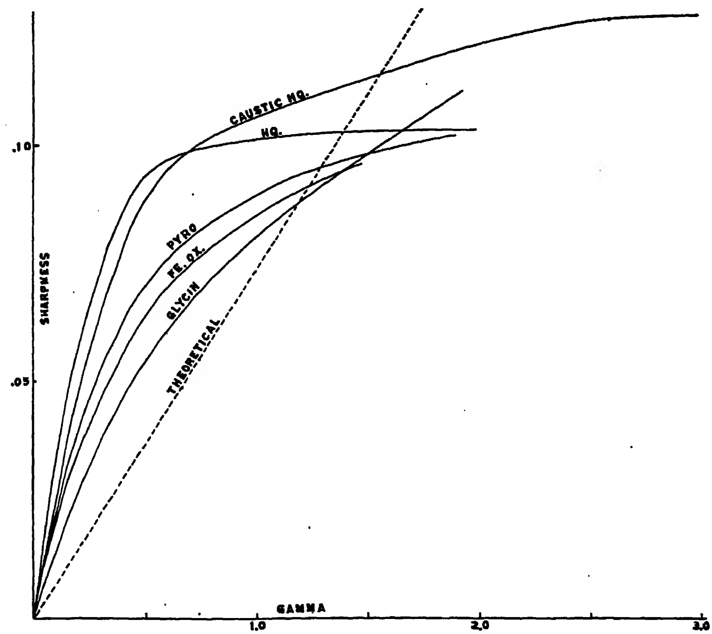


FIG. 55
Theoretical and measured sharpness

of course a straight line through the origin. The interesting fact is brought out that for all developers the observed sharpness is greater than the theoretical value when gamma is less than about 1.2, and is less than the theoretical value for higher values of gamma. A possible qualitative explanation of this is given on p. 141. One cause of the differences found among developers, as shown by the curves, may be found in the Eberhard effect, which may also be looked upon as playing a part in the above-mentioned deviations from theory. A note-

worthy fact deduced from the curves is the relatively high values of the sharpness in the regions of low gamma which are given by developers containing hydroquinone It should be said that these curves are only provisional and are to serve as a basis for a more complete investigation of the dependence of sharpness upon developer and upon the various factors which control development

Edge Gradient Depending on Light Intensity It has been found that the edge gradient does not depend on the exposure time, except for great over-exposures and for very light exposures In the latter case, on account of the difficulty in measurement, this proposition is not at all certain In order to determine the "intensity" effect, two series of exposures were made on the same plate to the same light source with and without neutral film or opacity Light of various wavelengths was employed The results are summarized in Table 15 Each gradient is the mean result from 10 edges, each of which was measured twice Development was in caustic hydroquinone for 3 minutes, and accordingly complete The microphotometer magnification was 208, plate, Seed panchromatic

TABLE 15

Dependence of Sharpness on Intfnsity

λ	420μμ		450μμ		470μμ		520μμ		660μμ	
Opacity	0	25	0	625	5	125	0	625	0	125
Edge Gradient	154	126	131	113	087	089	071	073	106	093
Per cent Increase	+22		+16		−2		−2		+14	

The exposure times were chosen to produce densities as nearly equal as possible The results show a strong intensity effect in the violet and blue-violet, and a similar effect in the red Strong violet light gives a sharper image than weak violet light The data are not sufficient to determine the exact cause Sections through the image should be made and studied, and the corresponding values of gamma and turbidity determined The effect is probably due to the high opacity of the emulsion to violet light, combined with failure of the reciprocity law, producing in the case of the stronger light an abnormally greater thickness of developed image It is easily seen that other things being equal the sharpness of

an image considered from the standpoint of density gradient is directly proportional to its thickness In the case of resolving power this does not appear to hold, the thinner deposits giving the best resolution On p 149 it is shown that higher resolution is obtained with diminishing light intensity, so there does not appear to be any real conflict of data Sharpness and resolving power are thus seen to have points of difference, which will be considered in detail in a later paragraph

Sharpness of Images on Yellow-Dyed Plates Seed panchromatic plates bathed in yellow dye were exposed, in turn, in the monochromatic illuminator in the usual way to light of three wave-lengths, seven images in each case being obtained with an exposure range of 1 to 16 The mean results, as well as those obtained for comparison on undyed plates of the same kind, are given in Table 16

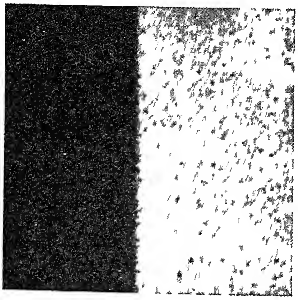
TABLE 16

INCREASE OF SHARPNESS WITH DYEING

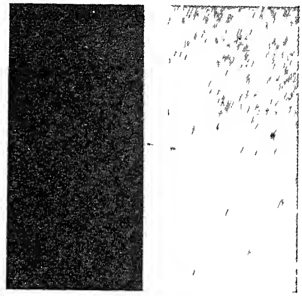
	Density Gradient		
Wave-length	420μμ	520μμ	660μμ
Dyed	0 220	0 079	0 086
Undyed	0 152	0 064	0 092
Per cent increase on dyeing	+45	+24	−7

These results are in harmony with what one might expect For yellow dyeing increases the opacity for blue and violet light, both vertically and horizontally In case of the shorter waves, the horizontal spreading (turbidity) is diminished to a greater extent than is the penetration so that greater sharpness results The small difference in the case of red light is probably without significance

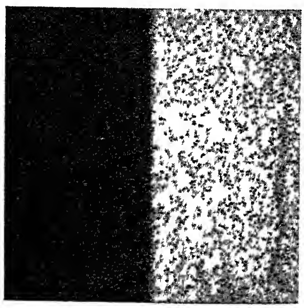
Sharpness of Images on Fine-Grain Plates No serious attempt has been made to measure the sharpness obtainable with the finer grain plates In order to make these measures successfully, a much higher magnification, from 500 to 1000, should be used The maximum sharpness obtainable will be found perhaps for the albumen plate A knife-edge contact print was made on an albumen plate which is shown in Fig. 56 under two magnifications 100 and 500 For comparison, an edge printed on Seed 23 plate is shown at the lower magnification The extraordinary sharpness of this plate is manifest For its resolving power, see pp 118 and 121



Albumen Magn 500



Albumen Magn 100



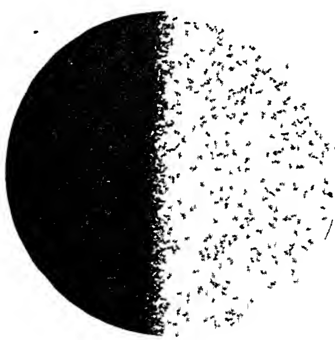
Seed 23 Magn 100

FIG 56

Enlargements showing sharpness on albumen and Seed 23 plates

Perhaps the most important of the series of measurements so far attempted with the microphotometer has been the determination of the sharpness of images made with light of different colors, or, in other words, determination of the curve

4000 A



5000A

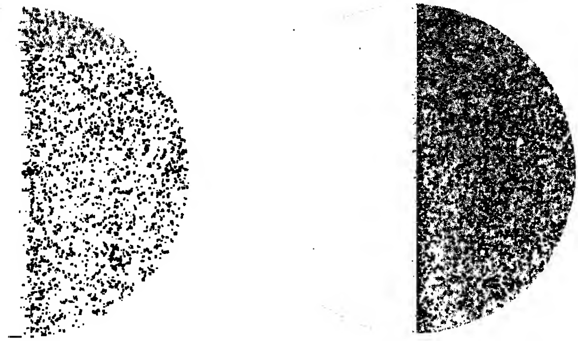


FIG. 57
Enlargements showing variation of sharpness with wave-length

of sharpness-variation with wave-length. Fig. 57 shows a series of edges printed by contact in the manner already described, using panchromatic emulsion, exposure being to light of wave-length 4000A to 7000A. The variation of

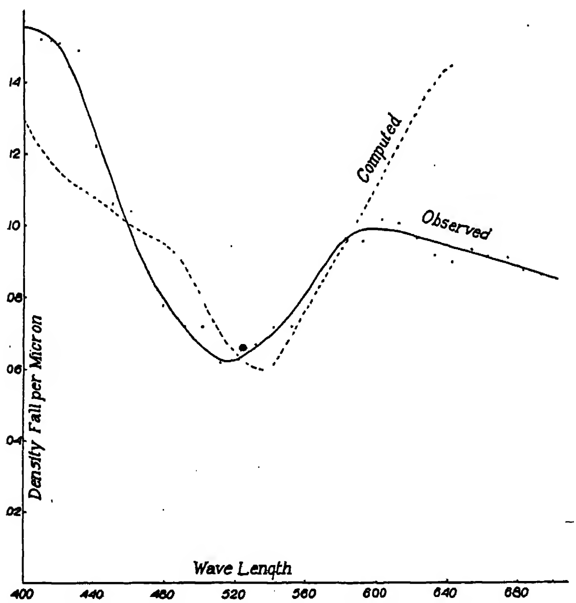


FIG. 58
Sharpness—wave-length curve

sharpness is strikingly apparent The original images, of which there were ten at each wave-length, were measured on the microphotometer The resulting sharpness, expressed as the fall of density (diffuse) in a distance of one micron, is shown in Fig 58 The sharpness wave-length curve computed from formula $S = \gamma \kappa$ is also shown In order to obtain the computed curve the wave-length turbidity and wave-length gamma curves are necessary These were accordingly measured, with the result shown in Fig 59 While the agree-

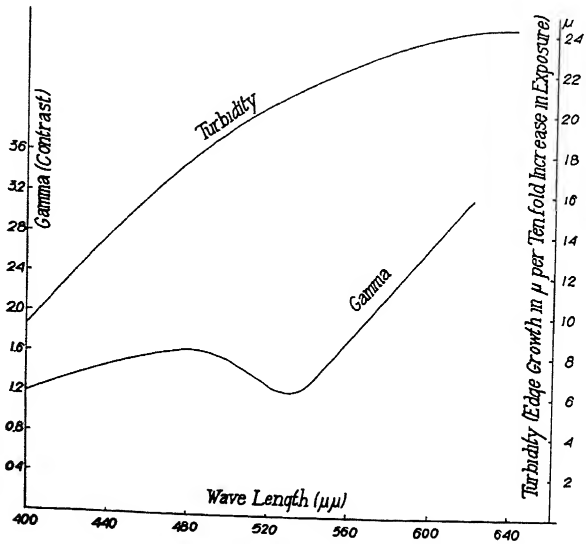


FIG 59
Turbidity and gamma curves depending on wave-length

ment between the computed and observed curves is not all that could be desired, it is sufficiently close to show that the general theory of the subject which has been developed is substantially correct, the differences being due to secondary causes, several of which have been already discussed

The wide difference between the two curves in the red is especially marked This is a point which requires clearing up, for which more data will be needed Determination of the contrast or γ in the red end of the spectrum appears for some unknown reason to be subject to very great uncertainty, variations of as much as 50 per cent being found in the same emulsion for definite wave-lengths in the red It is quite

robable that the disagreement of the two sharpness-curves i the red portion of the spectrum is related to this uncertainty r instability in contrast

Sharpness wave-length curves for a considerable number of mulsions were determined by R G Sherwood in this Laboraory, using the same apparatus as that employed by the writer The results of his determination for an isochromatic emulsion re shown in Fig 60 A large number of determinations were made, all of which are shown individually in the diagram as dots Each dot represents the mean of measurement of all the edges on each plate, in general about ten in number It is seen that the probable error of a determination of sharpness from the diversity in position of the dots, is roughly five per cent, a value not unduly large

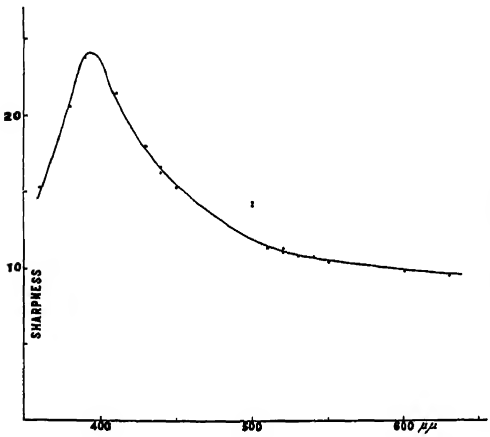


FIG 60
Sherwood's measures of sharpness

It will be useful to give the details of one series of measurements of sharpness of image The degree of accuracy to be expected in work of this kind will thus be made apparent A series of knife-edge contact prints on a standard emulsion was made with exposure to light of wave-length 4200 A Ten exposures were made on each of three strips The strips were developed for three-fourths minute, one and one-half minutes, and three minutes, respectively, in caustic hydroquinone The exposure time for the images on each strip was so chosen that the density developed to approximately unity in all cases The edge-gradient of each image was measured with the microphotometer at a magnification of 208 (the slits over the nose of the Martens photometer are 0 57 mm wide). Each edge was measured twice, at different portions of the edge There are thus twenty curves of edge-gradient for each development time The individual values are given in Table 17

TABLE 17

MEASURED EDGE-GRADIENTS S

Image	Three-fourth Minute's Development $\gamma = 0\ 71$		One and One-half Minutes' Development $\gamma = 1\ 16$		Three Minutes' Development $\gamma = 1\ 48$	
	First Measurement	Repeat	First Measurement	Repeat	First Measurement	Repeat
1	3 23	2 78	2 63	3 03	2 23	3 03
2	2 33	2 78	2 22	2 38	3 23	2 94
3	3 33	2 78	2 86	3 03	3 12	2 94
4	3 23	3 03	2 38	2 22	2 38	2 78
5	4 00	3 12	3 33	2 22	2 63	2 63
6	3 23	2 63	2 86	2 78	3 12	3 92
7	2 70	2 50	2 12	3 33	2 17	3 12
8	2 61	3 70	2 70	3 57	3 33	2 78
9	2 78	2 86	2 22	2 63	3 03	2 56
10	2 94	2 63	3 23	3 03	3 12	2 94
Means	2 96 ± 09 (p e)		2 73 ± 10		2 90 ± 09	

To reduce S in this table to absolute values (fall of density per micron), the numbers are to be multiplied by 0 0416 By comparing the individual values of S with the mean, the average error of a single determination of the edge-gradient is found to be $\pm 0\ 33$ or 12 per cent Repeats on the same section of edge show differences of only 3 per cent, indicating that the greater part of the error is photographic, that is, is due to photographic variations The most difficult case has been chosen for measurement, namely, that of an exceedingly sharp image of low density For the average edge, where the total fall of density is less abrupt, more accurate and consistent values are obtainable

As in the previous cases, it is possible to compare the edge gradients in Table 17 with theory The results are given in Table 18 Measurement gave $\Delta = 5\ 0\mu$, from which

$$S = 0\ 120\gamma \quad \text{(Eq 55}a\text{)}$$

TABLE 18

COMPARISON OF OBSERVED AND COMPUTED SHARPNESS

	γ	Observed S	Computed S
Three-fourths minute's development	0 71	0 123	0 085
One and one-half minutes' development	1 16	0 114	0 139
Three minutes' development	1 48	0 121	0 178

The computed values are seen to lie on both sides of the observed values The observed values are nearly constant, the computed values vary directly as γ The explanation of the discrepancy must be again sought in secondary phenomena Agreement is such that there is no doubt of the primary action being as postulated For γ unity or for normal development agreement between theory and observation is seen to be complete For lower development there is a secondary factor tending to increase sharpness, for higher development a factor tending to decrease it These will be considered in turn

In the case of weak development the silver grain forming the image is in an incompleted state This is seen very clearly in a microscopic study The image as a whole is grayish, compared with the dense black obtained with higher development, although the densities of the two images are equal Under the microscope, with short development, a large number of shadowy, indistinct grains are seen, which can not be focused They are due to partial development Grains are known to develop in spots, with numerous centers of development in each grain This process being arrested in short development, the result is a grain composed of gelatin imbedding scattering particles of metallic silver The grain therefore transmits and scatters the viewing light, causing both grayish and shadowy appearances, depending on its degree of development It is also seen that small grains are relatively numerous for short development This is as expected, on account of the time factor in completing development of a grain It is clear that short development must give a plate of relatively fine grain But the size of grain does not enter into the formula for sharpness, a statement which seems contrary to accepted views and to the measurements of sharpness now being discussed

There is other evidence indicating that silver bromide grains receiving a heavy exposure are more developable as well as more quickly developable than grains receiving less exposure Consider the case of a sharp image formed from a weak exposure receiving short development, as in the case under examination The latent image in the region bordering the true edge must be in this case relatively weak and is accordingly not developed when the time of development is short Sharpness must therefore be increased Longer or normal development ($\gamma = 1$), on the contrary, brings out the true latent image In this case the formula for sharpness is found to hold This explanation finds support when the case of very short time of development is examined For

example, when images on a plate developed for only twenty seconds are measured, it is found that the sharpness has decreased considerably This shows that the diminishing effect of a decreasing γ is becoming effective, more than counterbalancing the increasing effect due to non-development of the latent image

It is a common impression that small grain-size is the cause of sharpness of image It is true that in the case of resolving power, size of grain is an important factor But the matter of image sharpness is quite different This depends primarily as shown upon contrast and turbidity Usually fine grain plates have high contrast (γ) and low turbidity $\left(\frac{1}{\kappa}\right)$, which accounts for the grain being undeservedly given the credit for the sharpness or lack of it Sharpness due to uniform random distribution of silver grains, a popular conception of the subject, is of a higher order than occurs in real emulsions.

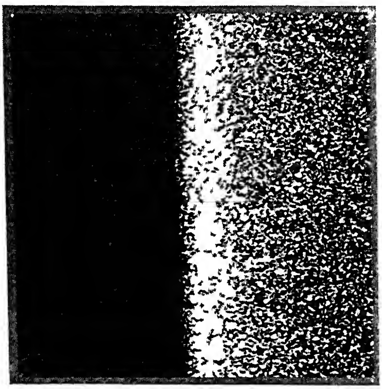


Fig 61
Edge of image showing absence of fog

The decrease in the theoretical sharpness for long development remains to be explained The cause is to be found in the phenomenon of graininess which appears to increase with development This subject has been fully discussed in Chapter II

Some cases of secondary phenomena taking place at the edge of an image are worthy of brief mention The phenomenon of an apparently luminous band at the border of a dense image, the Mackie line, well seen in moving pictures, is one, well brought out in Fig 61 showing an edge enlarged 100 diameters, in the immediate neighborhood of which the fog is almost completely absent Again, Fig 13, Chap III, showing sections of sharp slit-images, reveals an added mass of silver grains in the lower layers of the emulsion immediately under the edge itself These are both effects of developer, the luminous band being probably due to the inhibitory action of the reaction products of development on the formation of chemical fog, and the added density at the edge itself to the

easy access of fresh developer from the side. These are important and interesting phenomena, but cannot be considered at length in this place. Undoubtedly they are related to the Eberhard and Kostinsky effects, which will be treated in the following chapter.

Another secondary effect of a totally different class arises in the measurement of densities near the edge of a sharp image. It would seem that a given mass of silver at the edge of a large image should measure photometrically higher than when placed in the center of the image. The amount of light transmitted by any given small region depends upon its direct illumination and upon the amount of light diffused and reflected into it by the surrounding regions. At the edge diffusion is entirely lost on one side, so that less total light is transmitted, leading to a higher measured density.

PHOTOGRAPHIC RESOLVING POWER

So far as the writer is aware, the only formula which has been proposed for photographic resolving power is by Wadsworth (p. 108).

Study of the problem has led to a separation of the factors controlling resolving power into two, namely, (*a*) sharpness factor, (*b*) grain factor.

Consider a pair of parallel line images *aa* and *a'a'* on the photographic plate, of equal width, separated by a space w equal to the common width of the lines. Defining resolving power as the number of lines per millimeter just separated, we have

$$R = \frac{1000}{2w} \text{ (w in microns).} \tag{56}$$

It is not possible to derive any formula for R depending on physical data alone. It can be postulated that for resolution a certain contrast must exist between the brightness of the images *aa* and the center of the separating space b before resolution takes place. But brightness as a sensation is measured by density of the photographic deposit, so that it can be said that for resolution there must be a certain difference in density, which may be called J, between the images *aa*, *a'a'*, and the center of the separating space b. It is quite evident that J is a function of the diameter of the grain. For it is easily seen that if the grain of the plate is very fine, a relatively slight difference in contrast J between a and b will be sufficient for resolution. With increased size of grain, the

difference of contrast must increase The relation betwee size of grain and minimum contrast is to be regarded as belong ing to the class of physico-psychological laws of vision, obtai able as empirical relations by direct measurement, which a met at every turn in physical investigations in which visio plays a part It is perhaps needless to say that no dire data are available on the subject From general consider tions it is to be expected that the difference in contrast w approach a minimum value semi-asymptotically as the size grain approaches zero The simplest curve fulfilling t properties as imagined is probably the hyperbola, with tw arbitrary constants Accordingly,

$$J = (a+bd^2)^{1/2}, \qquad (5$$

where d is diameter of grain, and a, b are arbitrary constan to be determined by experiment It should be noted that is to a certain extent a function of the intensity of the viewi illumination, a complication not necessary to consider here

Let it be assumed that the fall of density from the edge of t image a to the center of the separating space b is uniform L S be the sharpness of the images aa or the rate of fall of densit as defined previously Let it be further assumed that t diffraction and aberration patterns of the two lines aa a $a'a'$ are separated This will be approximately true if t optical system is a good one With these assumptions it not difficult to derive a formula for resolving power, for definition

$$J = \frac{w}{2} S, \text{ or } w = 2\frac{J}{S}$$

Substituting for J and w their values in terms of size of gra and resolving power,

$$R = \frac{1000S}{4}(a+bd^2)^{-1/2}, \qquad (5$$

or, since $S = \kappa\gamma$,

$$R = 250\,\kappa\gamma\,(a+bd^2)^{-1/2}. \qquad (5$$

It should be possible by a series of suitably planned expe ments to determine the parameters a and b in this equatic This is a subject for future investigation It will be of inter to determine J in a particular case for which data are availab It was found, for example, that for a certain emulsion t resolving power was 70, determined from artificial dou stars at wave-length 4500 A The turbidity was found measuring the time-rate of increase of the diameter of artific

stars This was found to be 9μ for each doubling of exposure-time, i e, $\Delta = 9\ 0$, whence $\kappa = 067$ γ can be taken equal to unity For the determination of J, therefore,

$$R = 250\ \kappa\gamma\ J^{-1},$$

giving

$$J = 0\ 24$$

That is, for an emulsion of medium size of grain ($d = 2\mu$ to 3μ) the contrast necessary for resolution measured in density is equal to 0 24, or in light-intensities, since $D = \log \frac{T_1}{T_2}$, a ratio of 1 to 1 75 is necessary

Experimental results on resolving power have been studied with a view to the general application of the formula for resolving power deduced above They will be briefly considered

It is well known that fine-grain emulsions give higher resolution than those of coarse grain A particularly interesting case is the relative resolving power of an average emulsion, such as Seed 23, for chemical and physical development, respectively Obviously the turbidity factor $1/\kappa$ is the same in the two cases It is quite different, however, with the contrast factor γ For physical development γ was found to be 0 5, for chemical development, 2 5 Since sharpness $S = \kappa\gamma$, it would be expected that the sharpness under physical development would be very much less (five times) than under chemical development This is actually found to be the case, the difference being strikingly evident to the eye, without measurement Since the sharpness of outline is so much less for physical development, it might be expected that the resolving power would likewise be less As a matter of fact it was found that the resolving power under physical development is actually higher by 20 per cent We have here an interesting case of the balancing of the factor of sharpness against that of the size of the grain In physical development the grains are exceedingly small, many times smaller than under chemical development The drop in resolving power due to a low factor of contrast is more than made good by the accompanying decrease in size of grain A further testing of the formula is possible from an examination of the large amount of data secured by K Huse (page 121) on the dependence of resolving power on developer and development time Huse finds that the resolving power, for increasing development time, rises quickly to a maximum, remains constant for a considerable interval, then drops Here

again it is a question of changes in contrast, and size of grain, including graininess, acting as opposing factors

Resolving Power for Parallel Lines Some preliminary experiments indicated that the resolving power for parallel lines was higher than that obtained by the fan test-object The cause is doubtless due to psychological factors (p 144) To obtain accurate data, the test-object shown in Fig 62 was used In order to determine if resolving power is solely a question of distance between centers of adjacent lines, and not dependent on the width of the lines themselves, within certain limits at least, one-half of the lines were made much narrower than the width of the intervening space It was found that the width of the lines was without influence It appears from Table 19 that the fan test-object gives results which are systematically low The fan test-object is shown on p 121

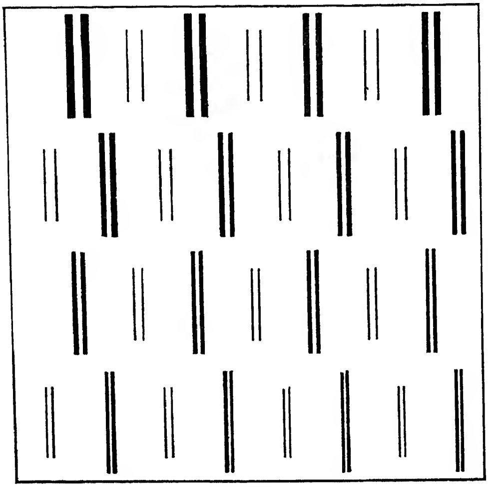


FIG 62
Test-object for resolving power

TABLE 19
RESOLVING POWER FOR PARALLEL LINES ON YELLOW-DYED PLATES

Plate	Fan Test-Object		Parallel Line (old dyed)	Per cent Gain
	Newly dyed	Old dyed		
Seed 30	61	68	99	46
Seed 23	72	86	125	45
Seed Lantern	87	108	140	30
Laboratory Lantern	84	103	140	36

In order to make a complete comparison of different methods of determining resolving power, tests with a double star test-object should be included Such a test-object has been made

and preliminary measures made which indicate agreement with results from the parallel line test-object. Fig. 63 shows enlargements of fan images. Fig. 64 that of an image made with the double star test-object.

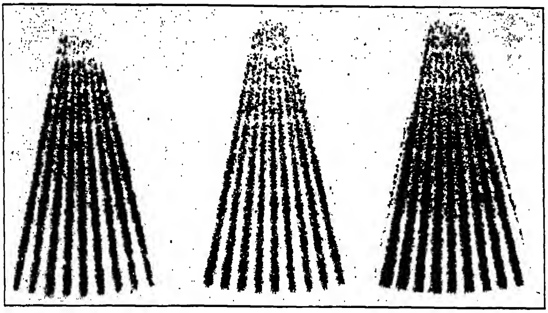


FIG. 63
Enlargements of images of fan test-object

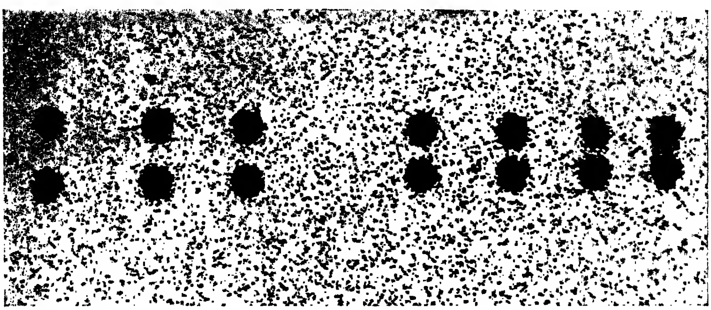


FIG. 64
Enlargement of an image of double-star test-object

Referring to Table 19, it will be noted that the resolving powers in the third column are 20 per cent higher than those of the second column. The plates used are, respectively, ones which had been dyed two months previously, and ones freshly dyed. This indicates that the resolving power of yellow-dyed plates increases with the time after dyeing. The cause is not known. It was noted, in addition, that the speed of the plates diminished with age, the loss being 50 per cent in the first day after dyeing. The two phenomena are undoubtedly connected.

It is commonly supposed that the gain in resolving power of plates after bathing in yellow dye (acid yellow) is due to the higher absorption of the dye for the blue and violet rays, so that the image lies more on the surface and is, in addition, scattered less sideways This, however, cannot be the complete, or possibly even the main reason, for acid yellow is a desensitizer, and in general, all desensitizers, including the blue copper sulphate and the yellow chromic acid, if applied to a photographic plate, will increase its resolving power The improved resolution cannot be due to desensitization of the coarse grains, since it has been shown that the coarse grains are the most resistant to desensitizing It may, on the contrary, be due to the narrowing of the *range* of grain-sizes available, and to a steepening of the gradation In the case of the yellow desensitizers, increased opacity is undoubtedly a factor For example, it has been found that bathing in a 0 5 per cent solution of chromic acid increases the opacity to blue light ten times

In order to determine the effect of dyeing on the resolving power for different colors, exposures were made on dyed and undyed plates behind blue, green and red filters The following table summarizes the results

TABLE 20

Wave-length Effect on Resolving Power with Yellow-Dyed Plates

Plate	Blue Filter No 70 mean λ = 440			Green Filter No 62 mean λ = 540			Red Filter (F1) mean λ = 660		
	Undyed	Dyed	% Gain	Undyed	Dyed	% Gain	Undyed	Dyed	% Gain
Seed 30	42	61	45	26	34	31	42		
Seed 23	51	72	41	36	44	22	45		
Seed X Ray	50	70	40						
Seed Panchromatic	50	69	38	37	44	19	46	44	−4
Seed Lantern	57	87	53						
W & W Process Panchromatic	56	84	50	48	60	25	55	59	+8
Laboratory Lantern	57	85	49						

The gain in resolving power in the green is seen to be one-half that in the blue, while there is no gain in the red These results are not entirely explicable, either from a desensitization or an absorption view point Acid yellow is a desensitizer for the short waves only, a property not common to all desensitizers It is apparent that much work remains to be done in this field

Effect of Intensity on Resolving Power The effect of the specific brightness of an image upon its turbidity was consid-

ered on p. 100. The decreased turbidity found for lower intensities would indicate a corresponding increase in resolving power if turbidity is a factor of importance. To test this, a number of plates were exposed to the fan test-object in the resolving power camera, first with high intensity light, then with low intensity, the ratio being 650 to 1. The exposure times were chosen to secure the best resolution in each case. Results follow in Table 21. The gain in resolving power for low specific intensity is decisively shown.

TABLE 21

RESOLVING POWER FOR HIGH AND LOW INTENSITY

Plate	High Intensity	Low Intensity	Per Cent Increase
Seed 23	53	59	11
Seed Process	41	47	15
Standard Slow Lantern	47	54	15
Laboratory Lantern	41	48	17

TABLE 22

EFFECT OF DEVELOPER UPON RESOLVING POWER

Developer	Measured by		Mean
	Norris	Ross	
1 Metol-Hydroquinone	131	136	133
2 Hydroquinone—Metol Substitute	131	128	130
3 Aeral (Paradiaminophenol)	131	136	133
4 Metol-Hydroquinone	126	138	132
5 Pyro	129	136	133
6 Caustic Hydroquinone at 70°	128	142	135
7 Caustic Hydroquinone (1½ m. at 65°)	126	142	134
8 Caustic Hydroquinone (2 m. at 65°)	114	138	126
9 X-Ray Developer	120	132	126
10 Hydroquinone (4 m. at 68°)	116	123	120
11 Hydroquinone (8 m. at 68°)	123	134	129
12 Brady's X-Ray (1½ m. at 68°)	115	125	120
13 Pyro-Metol (acid fixing bath)	121	126	123
14 Pyro-Metol (plain fixing bath)	124	138	131
15 Caustic Pyro	125	119	122
16 Rodinal (1-10)	125	125	125
Mean	124	132	128

Effect of Developer upon Resolving Power. This subject has been thoroughly investigated by Huse (see p. 123), who found a pronounced dependence of resolving power upon the particular developer used. His tests were made with the fan test-object, using Seed Lantern plates. The series of determinations, the results of which are summarized in Table 22, were

made with the parallel line test-object, using yellow-dyed plates (cine positive emulsion) Each value given is the mean from eight separate images In order to show the influence if any, of the observer, the mean estimates of two observers are given An aesculin filter was used, which absorbs the out-of-focus ultra-violet light

The last two developers are those giving the highest and nearly the lowest values in Huse's experiments (p 123), the difference found by Huse being 57 per cent From Table 22 it is seen that the maximum deviation from the mean is only 7 per cent The conclusion is that under the special conditions chosen, *i e*, of emulsion and test-object, there is little or no dependence of resolving power upon developer The reason for this strong deviation from Huse's result is not known Theoretically, it would seem that the former result is quite correct, namely, that there should be a dependence of resolving power upon developer, since the latter is known to influence many of the factors upon which resolving power has been shown to depend In order to decide the question more experiments are necessary

The personal equation between the observers is well shown in Table 22 Its mean value (6 per cent) is, however, of little importance In judging resolution in the case of double stars, however, it is quite likely that the personality factor would be much larger than in the above, which is based on a parallel-line test-object

In order to show the structure of the image in the case of good resolution a section of film was cut across the fan image at a point where the resolution was 25, *i e*, the lines are 0 04 mm apart from center to center Yellow-dyed cine positive film was used It is seen (Fig 65) that the image is formed only in the upper half of the film Deep images, corresponding to full exposure or development, are fatal to the best resolution

An interesting case is that of the resolving power of the wet collodion plate It is well known that the size of the grain of the collodion plate is subject to control in the development operation By using nearly monochromatic violet light and by exceedingly careful focusing, in conjunction with the proper conditions of development, it was found pos-

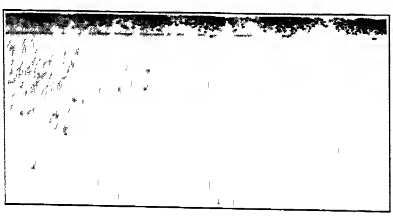


FIG 65
Enlarged section across an image of fan test-object

sible to obtain on this plate a resolving power of 175, or nearly equal to the theoretical resolving power of the lens used.

Fig. 66, A and B, shows the spectrum on collodion plates in an ultra-violet spectrograph, with and without an aesculin filter interposed. With the filter in, it is seen that the plate is

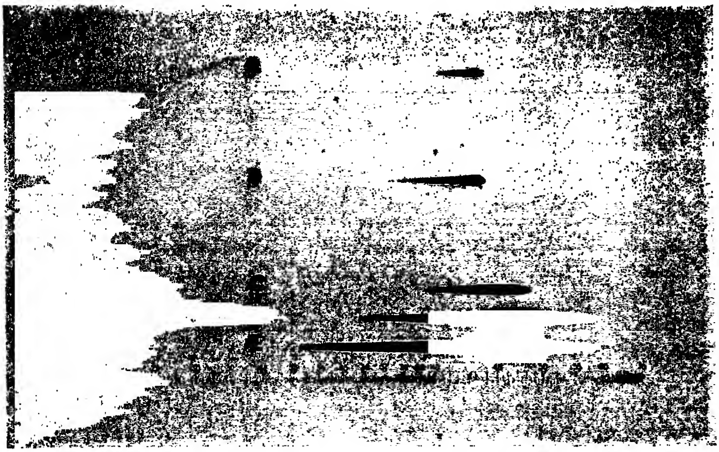


FIG. 66
Spectra on photographic plates, showing regions of sensitivity

sensitive to only a very narrow band of the spectrum, from 4000A to 4400A. Without the filter, the sensitivity reaches to 3650A. On account of the steep color-curve of even the best corrected lenses in the ultra-violet, it is important for the highest resolution to remove all light below 4000A.

It was found that intensification does not increase resolving power. It is useful, however, for it brings out the resolution more clearly when actually present.

Other secondary factors affecting resolution are: (*a*) image contraction, which tends to increase resolving power; (*b*) an effect pointed out by Goldberg[1] due to the difference in the sensitivity of the grains composing a high-speed emulsion. Briefly stated, he considers that an irregularity of outline of an image leading to diminished relsoution may be produced by numbers of highly sensitive silver bromide grains lying just without the edge of the image which are infected and made developable by the comparatively weak light to which they

[1] Goldberg E., l. c., p. 315.

are exposed, combined with numbers of insensitive grains lying just within the edge of the image too insensitive to be affected even by the strong light to which they are exposed It will be noted that both of these effects if operative modify the sharpness of image as well as resolving power, so that equation (58) for resolving power holds if the observed S and not the theoretical sharpness $\kappa\gamma$ be used

In applying the results obtained for sharpness and resolving power to a particular problem caution must be used From the fact that increased sharpness and resolving power have been secured by using violet light, and again by using yellow-dyed plates, in the particular experiments described above, it must not be assumed that the results are of general applicability For, as shown, sharpness and resolving power are directly proportional to gamma, and inversely proportional to turbidity Turbidity is decomposable into two parts (1) the inherent turbidity of the emulsion, (2) the turbidity arising from the optical system The results obtained above were secured by making the turbidity due to the optical system zero or nearly so Under average conditions of astronomical photography, due largely to atmospheric dispersion and unsteadiness, the optical turbidity becomes very large, completely overshadowing the inherent turbidity of the emulsion In this case then, gamma becomes the determining factor Plates of high contrast should accordingly be chosen to secure maximum sharpness Again, since it has been shown above that the contrast is low in the violet, and increases with increasing wave-length, it is apparent that isochromatic plates used with a yellow filter will give greater sharpness and resolving power in astronomical work This is actually found to be the case Still better results should be secured by photographing in the red In all cases of this kind where the optical turbidity predominates, the gamma-wave-length curve of Fig 59 becomes the true sharpness-wave-length curve in place of the curve shown in Fig 58, which holds only for an ideal or practically perfect optical system

The reason why yellow-dyed plates are not suitable for astronomical work is clear Such plates owe their high resolving power to a low inherent turbidity which, however, is of no help in the cases arising in astronomy But yellow-dyed plates have low contrast, or gamma, which accordingly makes them even less suitable than ordinary plates for astronomical use On the other hand, photographs taken on yellow-dyed film in a motion-picture camera having a good lens, have shown greatly improved definition over those taken on un-

dyed film. The principles enumerated above thus appear to be well founded and agree with practical findings. Since the relative amounts of inherent and optical turbidity cannot be specified in any given class of photographic work, it will always be necessary to make an experimental determination of the conditions giving maximum sharpness and resolving power.

Astronomers often ask for a high-speed plate of fine grain. Is not the advantage of small size of grain overestimated? For instance, the best linear resolution of double stars obtainable with large telescopes under the most favorable conditions is about 60 μ. The linear resolution obtained in the laboratory for emulsions of coarse grain and high speed is 20 μ. The true resolving power of the plate is thus far ahead of that which can be obtained in work with large telescopes. It is quite certain that an increase in the speed of plates will increase the resolution obtained in the large astronomical telescope, owing to a decrease in the effect of atmospheric tremors. It is not so clear that an increase in resolving power will work in the same direction to any appreciable extent, but it is at least true that increased speed should be attained without any loss of the resolving power which the plates now have. In other words, it is more profitable to work in the direction of attaining higher speed than attaining better resolving power.

The subject of resolving power can be partly treated in a descriptive and more popular manner which is not without helpfulness. Consider a pair of close lines being photographed with continually increasing times of exposure, starting from threshold. Resolution is not attained until a certain density of the image has been built up. When the exposure is relatively light only a few scattering grains are affected, so that there is little or no contrast between the image and the surrounding field fog. Two close lines under these circumstances will not be distinguishable. Let the exposure be now increased. The vacant spaces between the scattering grains, which lie at random where the image should be, begin to fill in with grains, the lines begin to outline themselves and finally, when a certain point has been reached, are clearly resolved. This is a description of what happens in the ideal case where there is no spreading or turbidity. In an actual emulsion, in the case of a line or star image, spreading begins before any relatively high density is reached, so that before a fine line can outline itself with sufficient distinctness, spreading has commenced, filling in the space between the two close-lying lines to a greater or less extent, thus limiting the resolving power

which can be attained. This process is observable and has been studied on ordinary lantern plates and on yellow-dyed lantern plates. In these two plates, the size of grain is the same, and yet there is a very great difference in resolving power, as already explained. The test-object was photographed on them in turn with increasing exposure-times. With the ordinary plate, by the time the fine lines were filled in to a sufficient density to outline them well, the intervening spaces were also filled in with grains to such an extent as to destroy the separation. This filling in is due to turbidity and is seen not to be connected with the size of grain.

Compare this with what happens in the case of the yellow-dyed plate. With light exposures only a few scattering grains are visible and there is no appearance of an image. On increasing the exposure the lines begin to fill in until they are finally well outlined, and yet the spaces between remain free from grains, in short, the image builds itself up without spreading. Accordingly, neglecting the factor of the size of grain we can say that what happens is this: in an emulsion of poor resolution by the time the image is built up to a sufficient density to outline it, the spreading is so extended as to destroy resolution through the filling in of the spaces between the lines, in an emulsion of good resolution the image is built up to a sufficient density to outline it before marked infection of the grains in the intervening spaces has taken place. Accordingly *turbidity and contrast of an emulsion are measures of resolving power*, a conclusion already reached by more mathematical reasoning.

PHOTOGRAPHY OF THE PLANETS

One of the most important applications of the subject of photographic resolving power is in connection with the photography of the surface of the planets and of the moon. In general, the results obtained have been of inferior quality compared with direct vision with suitable optical equipment. This is due largely, not to any failure of the plate in rendering power, but to atmospheric tremors which displace the image on the plate during the time necessary to take the photograph. If the speed of the plate could be increased many times, greatly improved photographs would be obtained.

Laboratory tests of photographic resolving power or of detail-rendering power can not be directly applied to astronomical photography, neither qualitatively nor comparatively, as shown on p. 152. An important factor which has not been touched upon above, is the relative contrasts in the

detail to be delineated. In laboratory tests of resolving power the contrast between light and shade in the detail is very high, the value varying widely, depending principally on the lens-system. In nature the contrasts in fine detail are not nearly so great. It is, accordingly, important to obtain some idea of the dependence of resolving power upon relative contrast. For this purpose a test-object was made consisting of dots, all of the same size, arranged in three concentric circles, dark on a bright background. The relative contrasts of the dots on each circle with respect to the common background were 1.23, 2.76, and 9.8. The computed size of the dots on the photographic plate in the precision reducing-camera was 35μ. A fine-grain emulsion in the camera was barely able to render all the dots. An ordinary fast emulsion showed only the two more contrasty circles. Upon doubling the size of the test-object, the size of the dots increasing to 70μ, the fast plate was able to show all of the dots. A photomicrograph is

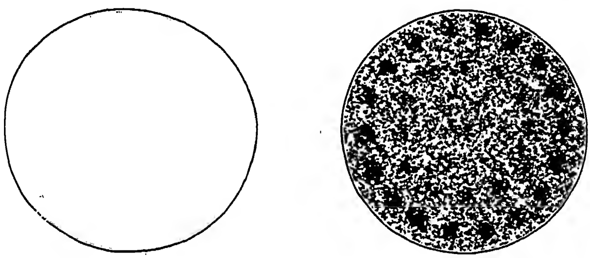


FIG. 67
Resolving power depending on contrast

shown in Fig. 67. Numerical results are collected in Table 23.

TABLE 23

	Resolving Power		
Contrast	9.8	2.76	1.23
Fine-grain plate	65	28 (est.)	14
Ordinary fast plate	35	14	7

In photographing bright detail upon a dark background, lines, for example, if the contrast is very high (more than 10, say) the size of the detail or the character of the plate is unimportant. Thus, the stars can be photographed under all conditions, given sufficient exposure. The case is quite different for dark detail on a bright background, for which the character of the plate and the size of the detail is of the greatest importance, as just shown. It will be of interest to show

results obtained in this last case, on account of its application to the photography of dark planetary markings A test-object was made consisting of a number of dark areas connected by fine lines, some double, the contrast between lines and background being very high The result of photographing on a fast and a slow plate is shown in Fig 68 The fast

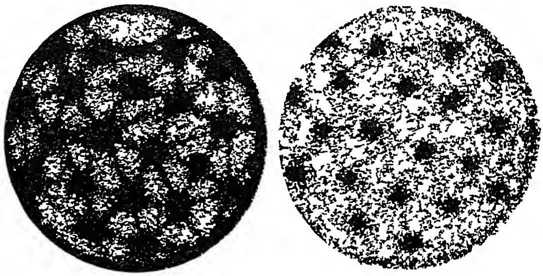


Fig 68
Resolution tests on planetary test-object

coarse-grain plate misses nearly all of the lines, only a few traces occasionally appearing, while the fine-grain plate shows the lines clearly In all cases the true width of the lines was falsely rendered, being three times too great, which shows the danger of judging size of detail from the corresponding size of its image on the photographic plate This holds for dark detail of high contrast on a bright background For reverse conditions the error would be much greater In the case of the experiments with the dots of low contrast, Fig 67, the true size of the detail was correctly shown on the plates In this case the detail was larger and the contrast less On account of the low contrasts on the planets it is quite likely that the size of small detail is correctly rendered For the moon, where the contrast in very small craters is very high, the size is likely to be overestimated

CHAPTER V

The Mutual Action of Adjacent Images

On account of the high degree of precision which had been reached in the determination of the positions of the stars by photography during the first decade of its application to this field, it was supposed that the photographic method could be employed with equal advantage in the determination of the relative positions of double stars and satellites. In both of these special fields of observational astronomy, the systematic errors in the usual visual methods are both troublesome and important, so that it was hoped that photographic methods might be employed with profit. However, without any question being raised at the time as to the accuracy of the photographic results, it was at once found that its application to the measurement of double stars was limited to pairs of only moderate separation, the important pairs having a separation of the order of one second of arc or less being unresolvable even with the largest telescopes. A greater field of usefulness appeared in observations of satellites, in which it was important to eliminate the systematic errors known to be present in the visual observations.

The first to question the accuracy of photographic measures of double stars and satellites, so far as the writer is aware, was S. Kostinsky[1] in 1906. He observed an apparent repulsion of neighboring images, and cites as a possible explanation the well known fact that in the wet condition images lie below the surface of the surrounding gelatin. In a later paper[2] Kostinsky discusses the phenomenon at length. By photographing a wide double star for varying exposure times, he finds a progressive retrocession of centers, as the following data, taken from his measures, show:

TABLE 24

Exposure Time	Diameter		Distance between Borders	Measured Distance between Centers
	Star	Companion		
min.	mm.	mm.	mm.	mm.
1	0.282	0.039	+0.086	0.2467
16	0.482	0.080	−0.014	0.2667

Thus, for the long exposure, giving an overlap of 0.014 mm., the distance relative to that for the shorter exposure increases

[1] Kostinsky, S., Mittheilungen der Nikolai-hauptsternwarte zu Pulkowo. Vol. I, No. II.

[2] Ibid., Vol. II, No. 14.

0 020 mm From all data he finds the apparent contraction, y, in microns

$$y = 7\ 9\left(1 - e^{\frac{80-x}{100}}\right),$$

where x is the distance in microns between adjacent borders of the images of the stars

H E Lau[1] in 1912 obtained similar results from a different procedure He exposed a plate to the Praesepe cluster for five minutes, then moved the telescope about 11″ and exposed again for five minutes A series of double stars of equal components but of a wide range in magnitude is thus obtained, which should vary in separation if the Kostinsky effect is present The resulting repulsion, y, as a function of the distance between borders, L, is shown in the following table

L	y
mm	mm
0 034	0 0027
051	0019
069	0008
086	0001
103	0000
120	0000
0 138	0 0000

Kostinsky's formula gives for L = 0 034, a contraction y = 0 0046 mm , which is somewhat larger than Lau's value

H H Turner,[2] on the other hand, notes an apparent *attraction* between adjacent lines on a photographic plate, *réseau* lines intersecting at a small inclination being bowed inward near the point of intersection The effect vanished when the centers of the lines were separated by ⅓ mm or more When the lines were in contact, the centers being then separated by 0 075 mm (computed), the actual separation was less by 0 006 mm This means that two fine spectral lines, each of width 0 075 mm , and in contact, have had their true separation diminished by 8 per cent Turner attributed the phenomenon to a displacement of the light maxima in the images, a phenomenon discussed at length below

F A Bellamy,[3] from a study of the double stars of the Oxford astrographic catalogue, supports Turner's conclusions Bellamy compares the photographic separations with the values in Burnham's *General Catalogue* In all, 436 pairs were examined In the case of the overlapping images (overlap-

[1] Lau H E Astronomische Nachrichten **192** 179 1912
[2] Turner H H Monthly Notices **77** 519 1917
[3] Bellamy F A Ibid **77** 521 1917

ping one-third of the radius) the apparent contraction was large, averaging about 8 per cent. With images not quite in contact, the clear space being 0.02 mm., the contraction amounted to 2 per cent. When the clear space became about 0.08 mm. all trace of contraction disappeared.

Hertzsprung[1] finds both contraction and repulsion in photographic measures of double stars depending on the atmospheric conditions and on the separation of the components. He explains the attraction as arising from the addition of density to each image at its inner edge, due mutually to the light from the other. This explanation is the same as Turner has suggested. He finds the effect most pronounced on nights of bad seeing, amounting to 0.007 mm. On the other hand, repulsions occur on nights of the best definition and in the case of close pairs.

Mitchell and Olivier[2] made an extended series of photographic measurements of the triple star Krueger 60, in which the components A and B approached within 1″.84 (0.09 mm.) of each other. Comparison with Barnard's visual measures shows no evidence of either attraction or repulsion.

There appears to be but little in the literature on the behavior of close spectral lines. C. E. St. John[3] has the following:

"A comparison has recently been made between the separation of close pairs as given in the Rowland tables and as measured upon plates of higher dispersion than that used by Rowland. For pairs consisting of lines of intensities 3 and 4 whose separations are 0 to 0.1A, 0.1—0.2A, and 0.2—0.35A, the Rowland values exceed those found at Mt. Wilson by 0.011A, 0.007A, and 0.004A, respectively."

From one point of view these can be considered as repulsions of the lines on the Rowland plates, the scale of which is 1A = 1 mm. The effect appears to be greater and present at greater separations than found in the case of stellar images described above. However, according to another investigation by St. John,[4] there is evidence of part of the effect at least being due to psychological causes. He finds that for measures of close spectral lines made in the ordinary way and on the Koch microphotometer, the microphotometer gives the smaller separation.

[1] Hertzsprung, E., Photographic measures of double stars. Observatory **44**: 56. 1921.

[2] Mitchell, S. A., and Olivier, C. P., Photographic measures of Krueger 60 as a double star. Astronomical J. **32**: 179. 1920.

[3] St. John, C. E., The situation in regard to Rowland's preliminary table of solar spectrum wave-lengths. Proc. Nat. Acad. Sci. **2**: 228. 1916.

[4] St. John, C. E., Year Book of the Carnegie Institution of Wash. No. 15. p. 240. 1916.

On the other hand, F Goos[1] finds that Kayser's measures of the separation of close doubles in the iron arc differ from his own in the following way

Average separation, 0 24 A=0 09 mm

Separation (Goos—Kayser),—0 0042A= —0 0016 mm ±0 0004 mm

Scale Kayser, 1 A=1 mm, Goos, 1 A=0 36 mm

Since the scale of Kayser's plates is the greater, these results show that the lines of Goos's plates suffer an *attraction*, in opposition to the repulsion shown on the Rowland plates Thus an inconsistency in the results affecting any general conclusions is to be found in measures of spectral doubles, as well as in measures of double stars

Aside from possible systematic errors of measurement, there appear to be three distinct factors controlling the deviations from normal action in the case of neighboring images, as follows

a) A mutual light-action causing each of the images to assume an oval form, the geometrical centers being displaced inward, the result being an apparent attraction This effect will be called the "turbidity effect"

b) Displacements of the gelatin in which the images are imbedded, which can be called the "gelatin effect"

c) It appears necessary to assume the existence of an abnormal action of the developer in developing the latent image, such as is found in the Eberhard effect[2] This abnormal repulsive action will be called the "Kostinsky effect" Each of the three phenomena will be considered at some length

The Turbidity Effect Consider the case of photographing a double star or a pair of spectral lines having components of equal intensity The distribution of light in the film due to each component is (p 93)

$$I=I_0 e^{-\kappa r}, \tag{60}$$

where I_0 is the intensity in the central diffraction disk, and I the intensity at a distance r from the center of star or line κ is a parameter whose numerical value is given by (p 105)

$$\kappa=\frac{2 \log 2}{\text{Mod}}\,\frac{1}{\Delta}=\frac{1\ 386}{\Delta}, \tag{61}$$

where Δ is the increase, in microns, in the diameter of one of the stellar images or spectral lines for each doubling of the exposure time, or, more strictly, for doubling of the intensity The relative distribution of light is thus fully determined

[1] Goos F Standard wave-lengths in the arc spectrum of iron reduced to the international unit Astrophys J 35 232 1912

[2] Eberhard, G l c

Let S_1 and S_2 (Fig. 69) represent the centers of a pair of close stars of equal components. Let P be any point in the immediate neighborhood, distant r_1 and r_2 from S_1 and S_2. The total intensity of light at P is evidently

$$I_p = I(e^{-kr_1} + e^{-kr_2}). \quad (62)$$

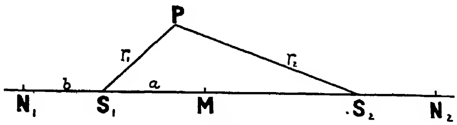


FIG. 69

The boundary line of the stellar image is determinable by the condition I_p = constant, which gives as the equation of its outline

$$e^{-kr_1} + e^{-kr_2} = \text{constant} = C. \quad (63)$$

This represents a family of transcendental curves, real branches of which are a family of lemniscates. In Fig. 70 a number of the curves are accurately drawn to scale. The practical case is chosen of a separation of centers of 0.1 mm. κ is chosen equal to 0.05, corresponding to Δ = 28, a value which is of the order of magnitude found for large telescopes and high-speed emulsions. In Fig. 70 a pair of point images are chosen, corresponding to the case of critical definition. In Fig. 71 the curves for disk-images are shown. In this case r_1 and r_2 are measured from the edge of each disk. It is seen that the character of the curves is the same in the two cases. The curves are oval in form, becoming more and more pronounced as the exposure time or intensity is increased. The center of each image is displaced toward the other, the amount of the displacement increasing with the size. In other words, double stars and close spectral lines are subject to an apparent *attraction* of an amount depending upon the

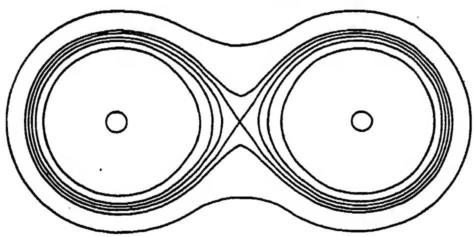


FIG. 70
Outlines of neighboring images for point sources

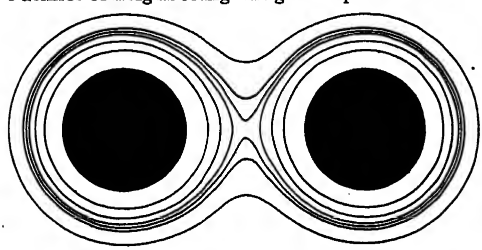


FIG. 71
Outlines of neighboring images for equal circular sources

separation of their adjacent borders In the important and interesting case of *contact* the figures show that instead of the images coming together along a curve as would naturally be expected, they meet in a point, the outline of each image in the neighborhood of their intersection being composed of two lines intersecting at an angle of considerable size in the practical cases under consideration These oval forms should be observable, and the corresponding attraction of centers should be measurable That they are not, in general, is due to the masking effect of the Kostinsky phenomenon, as will appear later

It is not difficult to obtain a formula giving the attraction or displacement of centers of adjacent stellar images or spectral lines Since the maximum effect is attained for equal components, only that case will be considered The method is easily extended to the case of unequal components Referring to Fig 69, putting $S_1S_2 = s$, equating the illumination at M with that at N_1

$$I\,[e^{-ka}+e^{-k(s-a)}]=I\,[e^{-kb}+e^{-k(s+b)}] \tag{64}$$

In all practical cases the second term on the right can be neglected The equation is then rearranged as follows

$$e^{k(a-b)}=1+e^{-k(s-2a)} \tag{65}$$

Put

δ = total displacement of centers,
r = separation of adjacent edges,

it is evident that

$$\delta=a-b,$$
$$r=s-2a,$$

leading to the following simple equation for the attraction of images of double stars or spectral pairs

$$e^{k\delta}=1+e^{-kr} \tag{66}$$

For the special case of contact of images, the equation becomes

$$e^{k\delta}=2,$$

or finally, since

$$\kappa=\frac{2\log 2}{Mod}\,\frac{1}{\triangle},$$

$$\delta=\tfrac{1}{2}\triangle \tag{67}$$

Hence we have the remarkable theorem that in the case of contact of images the contraction is equal to the increase which

takes place in the radius of a stellar image for each doubling of the exposure time, and is moreover independent of the size of the image itself. The theorem applies to spectral lines as well, in this case the radius being replaced by the half-width. In practical cases Δ lies between 10 (very sharp) and 30 (diffuse); the corresponding maximum attraction of stellar images or spectral lines accordingly lies between 0.005 mm. and 0.015 mm. Table 25, computed from (66), gives the value of the attraction δ for various values of Δ, and of r, the separation of the adjacent edges of stellar images or spectral lines.

TABLE 25

VALUES OF δ (IN MICRONS)

Δ	10μ	15μ	20μ	25μ	30μ
κ	0.139	0.093	0.070	0.056	0.046
mm.					
r=0.000	5.0	7.5	10.0	12.5	15.0
.010	1.6	3.6	5.8	8.1	10.6
.020	0.4	1.6	3.2	5.0	7.3
.040	0.0	0.3	0.9	1.8	3.2
.080	0.0	0.0	0.1	0.2	0.5

THE GELATIN EFFECT

In studying the relative turbidity Δ of large and small circular images (p. 98), Fig. 40, it was found that for the large image of diameter 1.72 mm., the initial rate of growth Δ was abnormally small, the normal rate not being reached until the exposure had become relatively large, in marked contrast to the behavior of small images, in which the normal rate of increase in size is maintained from threshold. In a study of the behavior of larger images, up to 5 mm. or more, an actual diminution in diameter was observed in the initial stages. The relative behavior of images in this respect is shown diagrammatically in Fig. 72, the curves shown being the result of actual measurements. The normal growth of an image with exposure is seen to be subject to interference by an amount depending on the size of the threshold or geometrical image. It may be inferred that there is a contracting influence present, which requires detailed examination.

The test-object shown to scale in Fig. 73 was made for the purpose of investigating the subject of the contraction of an image and its possible effect on the position of smaller or

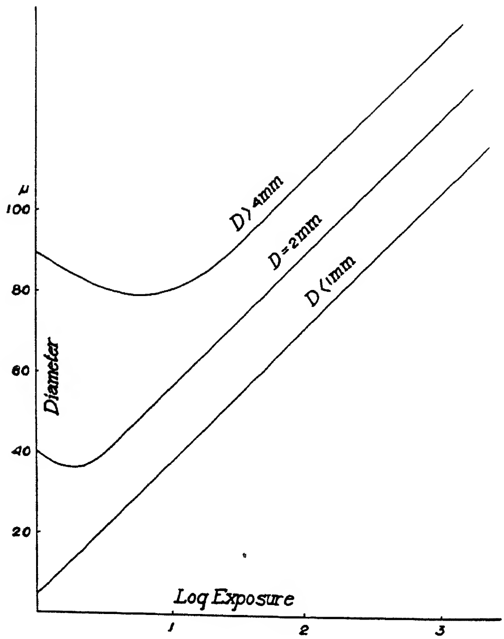


FIG 72
Dependence of turbidity on size of image

FIG 73
Test-object for image contraction and displacement

point images lying at varying distances from it It consists of a large circular hole 35 mm in diameter cut from cardboard and surrounded in the manner shown by smaller holes 1 mm in diameter A plug for the large hole was provided, so that exposures could be made with or without the large central

image Clearly this will enable the influence of the central image on the position of images lying at varying distances to be studied Two series of exposures were made on separate plates These plates are one inch wide and four inches long The camera is the one used in tests of resolving power, reducing 20 diameters Ten exposures were made on each plate in two groups, exposure-times being in all cases 4, 6, 8, 16, and 32 seconds The first group of five exposures was made with the central hole covered with its plug, the second group with plug removed One plate was developed in a pyro-metol developer, useful in development of aerial negatives on account of the stain produced, the other was developed in a chlorhydroquinone developer The fixing bath was without hardener Each plate was measured twice, first while in the wet condition, and again after drying When an emulsion is wet, it is necessary to measure through the back in order to eliminate errors due to capillarity In Table 26, which gives the result of the measures, *aa′* designates the distance between

TABLE 26

	Diameter of Disk mm	*aa′* mm	*bb′* mm	*cc′* mm	*dd′* mm	*ee′* mm	
			Plate I Developer, Pyro-Metol				
Disk out		2 078	2 684	3 362	3 981	4 628	Measured wet
Disk in	1 747	2 074	2 685	3 361	3 984	4 631	
Disk out		2 080	2 685	3 361	3 984	4 629	Measured dry
Disk in	1 638	2 033	2 673	3 357	3 979	4 625	
Disk out		− 002	− 001	+ 001	− 003	− 001	Contraction in drying
Disk in	+ 109	+ 041	+ 012	+ 004	+ 005	+ 006	
			Plate II Developer, Chlorhydroquinone				
Disk out		2 086	2 690	3 360	3 986	4 637	Meası red wet
Disk in	1 732	2 077	2 682	3 360	3.982	4 630	
Disk out		2 081	2 688	3 363	3 985	4 636	Measured dry
Disk in	1 720	2 082	2 688	3 359	3 981	4 632	
Disk out		+ 005	+ 002	− 003	+ 001	+ 001	Contraction in drying
Disk in	+ 012	− 005	− 006	+ 001	+ 001	− 002	

the pairs of images aa', etc Only the mean values are given, so that each quantity is the average of five measured quantities and has an uncertainty of not more than 0 001 mm All measurements were made on a Hilger comparator

This table discloses several facts (1) with the large central image absent it is seen to be immaterial whether the stellar distances are measured wet or dry, results from the two developers agreeing, (2) with the central image present, in the case of the plate developed in pyro-metol, there is a large contraction of the central image during drying, accompanied by a drawing together of the star images a and a' which lie near its edge, an effect which is seen to diminish rapidly as the distance from the edge of the central image is increased, (3) with chlorhydroquinone developer no certain effect of any kind is apparent

To study this phenomenon of apparent shrinkage, sections were made of images developed respectively in pyro and hydroquinone, which are shown in Fig 74, *d* and *e* The photomicrographs were made with the sections swelled with water They show a seriously disturbed condition with pyro development (74*d*), the image lying below the surface of the surrounding gelatin, while for hydroquinone development (74*e*), the image is slightly above the surface As further evidence of strong physical effects depending on development Fig 74*f* showing a double star is given There is seen to be a fold in the surface of the gelatin between the two components These disturbances are not peculiar to the wet condition Fig 74*c*

FIG 74*a*

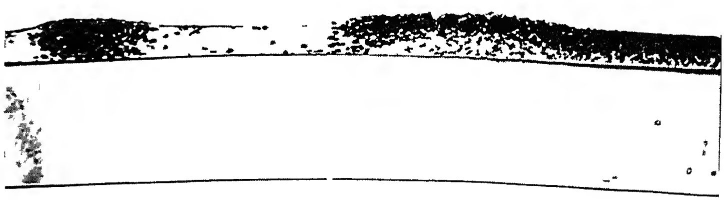


FIG 74*b*

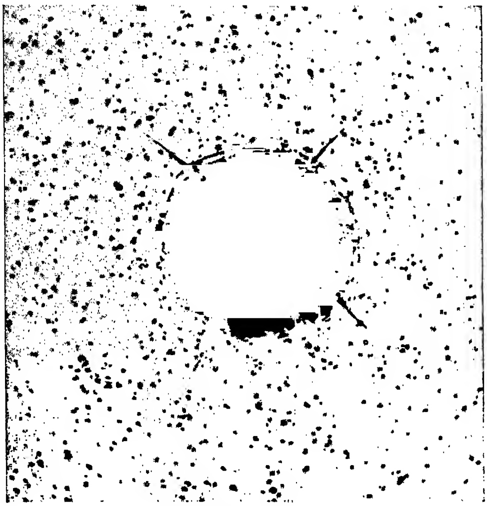


FIG. 74*c*

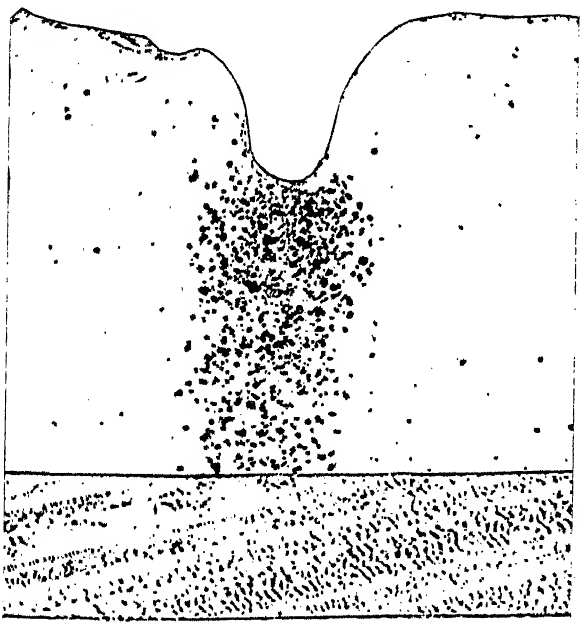


FIG. 74*d*

shows an enlargement of a star image, developed in pyro-metol, made when dry Ray-like lines are seen to radiate from the edge of the disk Furthermore, the entire edge of the image is seen to be in a disturbed state This is in line with results obtained in practice For in measuring diameters of artificial star images of great sharpness and density, such as are produced when development is in caustic hydroquinone, it is found to be impossible to focus sharply on the edge, showing that disturbed optical conditions prevail similar to that shown in the photomicrogram Fig 74*a* shows a section of a dry stellar image

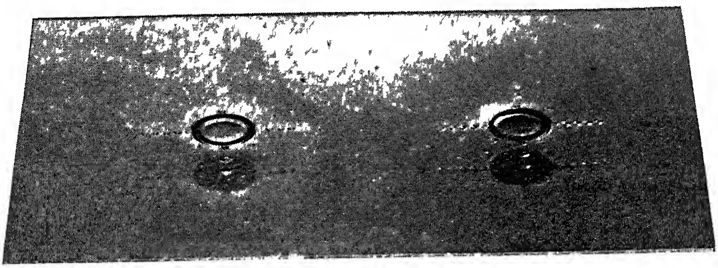


Fig 75
Perspective photograph of an image while wet

Fig 75 shows a perspective photograph of the surface of a plate which had been exposed twice to the test-object (Fig 73), the photograph being taken while the plate was still wet Note the large ring cavity at the edge of the image where it joins the gelatin, and the secondary central depression

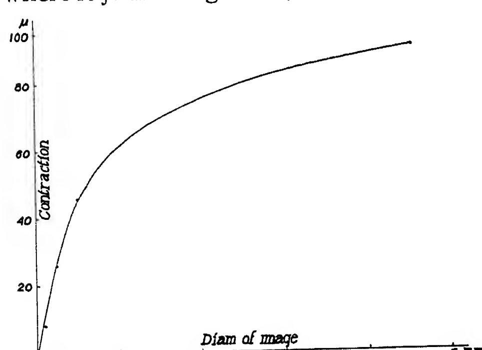


Fig 76
Relation between the contraction of an image and its size

Effect of Size on Contraction A series of images with diameters ranging from 0 1 mm to 4 5 mm was measured for image contraction The results are shown in the curve in Fig 76 The relation between contraction and image-size is seen to be exponential It may be inferred from this curve that a max-

FIG 74e

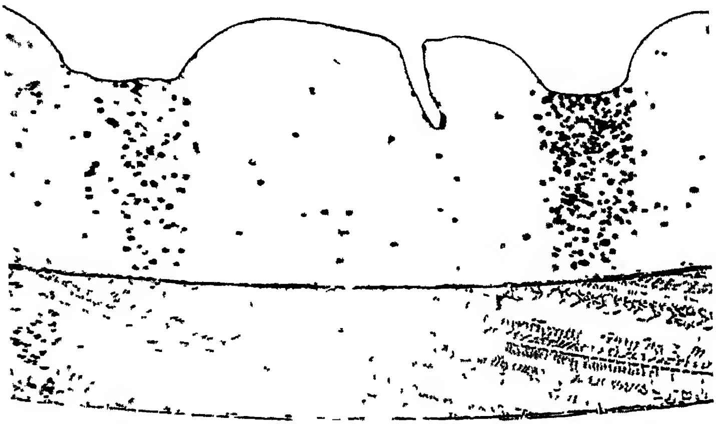


FIG 74f
Enlarged sections of images

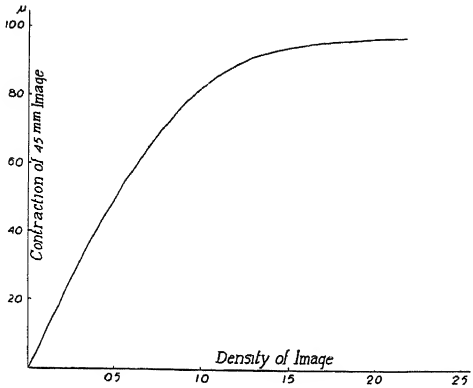


Fig 77
Relation between the contraction of an image and its density

imum contraction is reached for an image-diameter of about 5 mm, remaining constant for larger images This suggests that the disturbance is localized in a band about 2 5 mm wide just within the edge of the image, the larger part being within 0 6 mm of the edge That it is an edge effect is clearly indicated in Fig 74*b*, which shows a section of an extended image lying close to a star image The disturbance here is in a band 0 14 mm wide, which appears to be the region of most violent disturbance From Fig 76 the contraction is seen to be quite sensible for stellar images of sizes occurring in practice, amounting to 12μ per 0 1 mm diameter The normal growth of a star image under these circumstances is accordingly diminished 12 per cent through contraction

Effect of Density on Contraction The contraction of a 4 5 mm image of various densities from medium to high was obtained in a similar manner Results are plotted and the curve drawn in Fig 77 Here again an exponential relation is found between contraction and image-density

The curves in Fig 72 can now be explained For star images ($D<0$ 5 mm) the contraction is proportional strictly to the diameter, so that the only effect is a diminution of astrogamma Consider now the case of an image of 5 mm diameter, exposed to an increasing series of intensities At threshold there is a normal *increase* in diameter, due to turbidity, which, however, is no greater than for small star images But, as seen from Fig 72, the *contraction* is much greater than for small images, even at a low initial density There is, accordingly, an initial *decrease* in size which persists until the normal turbidity growth can make up the loss suffered by contraction The period of depression or of no growth is evidently a function of the size of image, with no change for diameters exceeding 5 mm

EFFECT OF GELATIN CONTRACTION UPON THE RATE OF GROWTH OF THE IMAGE

In the case of small disks such as are produced when stars are photographed the contraction should be uniform throughout the mass of developed silver, if the theory of the action proposed is correct, so that the amount of the contraction in the case of a series of star disks or images should be proportional to the diameters. Accordingly the only effect observable should be a reduction in Δ, the rate of growth. In order to test this and to find the magnitude of the change in the case of a number of developers, series of star images wereobtained in the usual way in the precision camera, exposures on each plate ranging from 1 to 240 seconds, so that Δ is well determined. Exposures were to white light (tungsten). The test-object contained three small circular holes ranging from 0.17 to 0.16 mm. in diameter, producing threshold images of which the smallest is 0.011 mm. Thus each plate gave three independent values of Δ. The plates were measured wet and dry, the measurements while wet being made through the back of the plate.

TABLE 27

Developer	Mean Diameter		Δ		Per Cent Decrease
		Wet—Dry	Wet	Dry	
	μ	μ	μ	μ	
Caustic Hydroquinone (fixed in plain hypo)....	77	7	11.3	10.4	8
Paradiamidophenol (Aerol) (fixed in plain hypo)....	82	6	12.3	11.4	7
Pyro-Metol (fixed in plain hypo)....	70	9	10.1	8.3	18
Hydroquinone (fixed in plain hypo)....	75	4	10.5	10.0	5
Pyro (fixed in acid hypo).....	69	7	10.8	8.8	18
Paradiamidophenol (fixed in acid hypo).....	75	4	11.0	11.0	0

A part of the difference found in Table 27 between the diameters, measured wet and dry, is probably due to personality in measurement. The expected decrease in Δ, as well as its variation with developer, is well shown.

It remains to consider the cause of image-contraction, its wide range with developer, and its possible dependence on other conditions, limiting ourselves to that part of it disclosed by

measurements "wet" and "dry," or to the assumed gelatin effect An indication of the immediate cause at least is to be found in the sections of star and of extended images already shown In the case of those developers giving greatest contraction, a strongly depressed wet image is found, while in the case of the developers giving little or no contraction a slightly elevated image is found That particular physical property of gelatin which is most strongly affected by chemical action is hydration, or ability to absorb water Sheppard and Elliott[1] have shown that one gram of gelatin which absorbs 50 grams of water under certain conditions as to acidity, will absorb but 10 grams when the acidity is only slightly changed

Consider what takes place when a photographic emulsion dries on a plate The natural tendency on dehydrating is to shrink equally in all directions, but on account of its strong physical affinity for glass it is unable to do so in a direction parallel to the plate As a result there is a residual stress in this direction which is so strong that it may even cause the plate to take a slightly concave form When an emulsion is stripped from a plate it will swell in its own plane only 25 per cent on complete hydration It is clear from this that its horizontal elasticity or ability to swell has not been entirely lost It is also clear that the physical structure of the gelatin has been profoundly modified by the stresses to which it has been subjected while drying on its glass support

The phenomenon of contraction of photographic images and distortions in general can now be explained On account of the smaller water content of the tanned developed image as compared with that of the surrounding gelatin, the image dries more quickly Stresses parallel to the plate, at the edge of the image, which for brevity may be called horizontal stresses, accordingly become unbalanced, since the normally active counterbalancing stresses acting outward from the edge of the image have not yet developed owing to the still wet condition of the gelatin in the region surrounding the image There is, therefore, a movement inward of the outer ring of the image The movement should be greatest at the extreme edge, diminishing gradually toward the center, a diminution which is due to the accumulating reinforcement of the counterbalancing stresses It has already been shown that this movement entirely ceases at about 2 5 mm from the edge At this point there must be a complete balance With the edge moving inward, the gelatin in its immediate neighborhood must be

[1] Sheppard S E and Elliott F A, The reticulation of gelatin J Ind Eng Chem **10** 727 1918

dragged along by an amount which appears to be an exponential function of the distance from the edge. This accounts for the translatory motion of star images already described, which has been found above to be large in the immediate vicinity of the edge, decreasing rapidly with increase of distance.

An interesting application is to close double stars. In this case the separate images are so close together that the intervening gelatin must be in nearly the same condition as to moisture content as the images themselves. So far as this phenomenon is concerned, therefore, the two images act as a unit, so that contraction must take place toward the mid-point of the separating space as a center. The distance between the two true centers must accordingly diminish. The same is true of close spectral lines, which ought to show the phenomenon also. It should be mentioned at this point that experiments were made to determine the effect of the form of an image on its contraction, with special reference to this matter of spectral lines. It was found that a long rectangular image 1.7 mm. in width contracted in width by the same amount as a square image measuring 1.7 mm. on a side. The contraction of spectral lines should accordingly be governed by their width alone.

The caustic hydroquinone developer frequently used by astronomers on account of the density and blackness of the silver deposits produced, gives strong contraction effects, and should be avoided in lines of work where extreme accuracy is required with no suspicion of systematic error. Its effect on clear vision of the edge of the image necessary in accurate photometric work has already been noted. This is true for pyro developer as well. Straight hydroquinone, metol-hydroquinone and chlorhydroquinone seem to give results in the main free from errors of contraction. A complete or extended classification of developers with reference to their contraction effect has not yet been made.

Lumière and Seyewetz[1] have investigated the tanning action of developers, especially pyro. They find that the tanning action of pyro is due to the rapid oxidation of the developer which takes place during actual development. The action is stronger at the regions of greatest density, where the oxidation is taking place most rapidly. Pyro will tan gelatin without development, provided air is present and sufficient time is given.

[1] Lumière, A. and L., and Seyewetz, A., The tanning of gelatin during development especially with pyro. Brit. J. Phot. 53: 285. 1906.

Their experiments with a number of developers "prove conclusively that when there is no tanning action with other developers as with pyro, the reason is that other developers only very slowly absorb atmospheric oxygen in the presence of sulphite As soon as they come under conditions which favor their oxidation, tanning occurs "

Also, "It may be assumed that the pyro is oxidized by the action of the bromine from the silver bromide, and that this oxidation product, which if formed in the presence of sulphite, makes the gelatin insoluble exactly in the same way as the quinone used in the free state did In regard to the other developers, in that they do not produce the insolubility of the gelatin when used under normal conditions, it is probable that their oxidation products formed during development are decomposed by the sulphite "

Contraction of Double Stars In order to determine to what extent the deductions of the preceding paragraph are correct, a test-object, *A*, composed of ten artificial double stars, all of equal size, was photographed in the precision camera on Seed 30 plates, the separations being adjusted so as to reproduce conditions obtained in practice An enlargement of one of these exposures is shown in Fig 77 The separation of the close pair (*jj'*) is 0 052 mm and of the widest pair (*aa'*) 0 104 mm The average diameter of the images is 0 04 mm Five exposures were made on each plate and were measured wet and

TABLE 28

CONTRACTION IN DRYING OF IMAGES OF CLOSE DOUBLE STARS (IN MICRONS)

Pair	Mean of Ordinary Developers	Pyro-Metol	Edge Separation
	μ	μ	mm
aa'	0 9	8 4	0 080
bb'	1 4	5 6	071
cc'	1 4	7 0	062
dd'	1 8	5 0	053
ee'	1 8	5 2	044
ff'	2 0	3 4	036
gg'	2 0	5 6	027
hh'	2 3		018
ii'	1 2		009
jj'	0 9		0 000±
Mean	1 6	5 7	

dry. The plates, ten in all, were developed in different classes of developers, including the staining: pyro, pyro-metol, and caustic hydroquinone; and the nonstaining: hydroquinone and metol-hydroquinone. Since the differences brought out between the action of the various developers, excepting the single case of pyro-metol, were not striking, the results from all ten plates are averaged in Table 28. All measurements recorded in this and in the following tables were made on the Hilger comparator at a uniform magnification of 75.

The values for the pairs i and j are uncertain owing to the difficulty of accurately measuring pairs in close contact. For this reason the three pairs which were measured last were not measured on the plate developed in pyro-metol. It is seen that the very strongly staining and tanning developer, pyro-metol, stands apart from the ordinary developers in the magnitude of the contraction, confirming the results found in the case of the disk test-object (p. 166). A further conclusion of importance is that the contraction is *independent* of the separation, within the limits of the experiment, which include all practical cases of interest. It can be concluded that, for the close doubles of separation 0.1 mm. or less, the contraction due to the movement of the gelatin is 0.0016 mm. for all ordinary developers. That different results were not obtained for the two classes of developers, staining and non-staining (with the single exception of pyro-metol), seems surprising in view of the results obtained with the disk test-object. It is clear that much work remains to be done on this branch of the subject before general conclusions as to the action of different developers can be reached.

The series of measurements on the double-star group furnishes information on the Kostinsky effect, as well as on the gelatin effect just considered, for measurement of the separation of the double stars on the test-plate itself, assuming no distortion in the optical system, gives the relative values of the separation of the images of the test-plate. For this comparison only the pairs aa' and jj' will be considered. The results are:

	mm.
Separation of centers of aa' in test-plate	2.000
Separation of centers of jj' in test-plate	1.022
Photographed aa' (measured wet)	0.1049
Photographed jj' (measured wet)	0.0532

By simple proportion from the first three quantities, jj' should be 0.0536 mm. This agrees with the measured value within the error of measurement, leading to the conclusion that no

distortion of any kind is apparent But from Table 25, assuming $\Delta = 10$, a value close to the true one, and assuming further that the images j and j' are in contact, we find a computed turbidity contraction of 0 0050 mm Since the gelatin effect has already been eliminated, it must be concluded that in this case the turbidity contraction and the Kostinsky repulsion are mutually destructive This is not surprising since attractions and repulsions in equal number have been obtained by various investigators, as previously pointed out

The Gelatin Effect for Spectral Emission Lines The test-object B in this case consisted of two pairs of parallel lines having equal components, which were photographed on Seed 30 plates in the same way as the double-star test-object The pairs are designated *a* and *b* in Table 29, which gives the results of measurement

TABLE 29

CONTRACTION OF SPECTRAL PAIRS IN DRYING

	a	*b*
Mean separation (dry)	0 149 mm	0 107 mm
Mean width of lines	0 100 mm	0 080 mm
Contraction with caustic-hydroquinone developer	5 0μ	4 3μ
Contraction with chlor-hydroquinone developer	4 6μ	4 6μ
Contraction with pyro-soda developer	6 2μ	6 0μ
Contraction with pyro-metol developer	15 4μ	14 3μ

It is seen that the gelatin contraction for spectral pairs is more than twice that for double stars of equal separation This may be in part due to the greater width of the spectral lines, as well as to their linear extent Further experiments with narrower lines are to be made It appears to be true that the gelatin effect is greater for the important case of spectral doubles than in that of double stars As in the case of double stars (Table 28), all developers excepting pyro-metol are seen to give nearly the same effect

Effect of Sulphite —Since the staining and tanning action of a developer to a certain extent can be controlled by its sulphite concentration, corresponding variations in the contraction of the gelatin should occur if it is true that tanning of the gelatin is a factor in contraction of the image. Experiments were made with pyro developer in which the amount of sulphite was altered between wide limits It is not necessary to give details The contraction was actually found to diminish with increasing sulphite

Action of hardening agents.—It is found that hardener in the fixing bath reduces the contraction only slightly. In a different experiment, plates after fixing were bathed in a concentrated solution of quinone; and in another case in a 10 per cent formalin solution. In order to obtain the maximum effect, the plates were developed in pyro-metol. An average reduction of 25 per cent in the contraction was obtained. These results were expected, since hardening of the gelatin should not alter to any great extent the variation of water content of a developed image with its density.

Drying Time of a Developed Image.—It is desirable to have direct or observational data on the drying time of a developed image, since in the explanation given of the contraction of an image due to movements of the gelatin it is assumed that a developed image dries more quickly than the clear gelatin surrounding it. The following experiments were made.

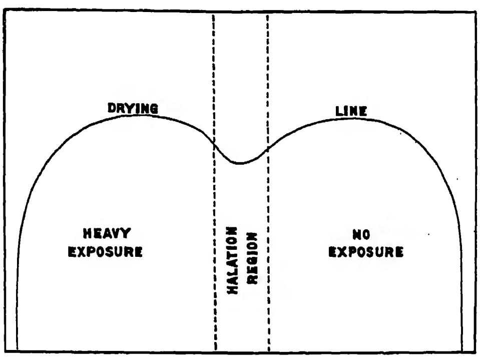


FIG. 78
Drying line on a partially dried plate

Five-by-seven plates were exposed to light over half their surface, developed in pyro-metol, and after fixing allowed to dry. The exposed portion was found to dry more quickly in all cases. Fig. 78 shows the drying line of a partially dried plate, one-half of which had been subjected to a heavy overexposure. Between the region of overexposure and the clear gelatin there is a halation strip one inch in width. The dip in the drying line in the halation region brings out the remarkable fact that a normal exposure dries more quickly than either clear gelatin or an overexposure. This difference in drying between normal and overexposure was verified by numerous experiments.

The Kostinsky Effect.—The anomalous actions just discussed which affect the true distance between images of close double stars and spectral lines on a photographic plate are not competent to explain the phenomena actually observed, for, as shown, both the turbidity effect and the gelatin effect result in an apparent attraction of images. As cited on p. 157, there

are many conclusive cases of apparent repulsion of images, so that we must look to a third class of anomalous actions to explain all the facts observed

One method of studying the mutual action of adjacent images is that followed by Kostinsky, namely, of measuring the distance of centers of a series of images of a double star or close spectral pairs, real or artificial, the time of exposure being varied in such a way that the size of the images varies from threshold to one producing contact or even overlapping When the experiment is made in this way, it must be assumed that the distribution of light in each image is perfectly symmetrical, or at least that any dissymmetry is the same for both images However, with good modern optical instruments a lack of symmetry need not be feared, if exposures are made on close doubles and in the optical axis In this connection it may be said that Lau's method (p 158) of obtaining the effect is open to question on this account, since the exposures were not axial, but distributed over the plate It is quite likely, however, that this error is negligible

Experiments with Artificial Double Stars—Two pairs of artificial double stars to be used as test-objects were accurately cut from a thin plate, the dimensions being as follows

Pair	Diameter of Circles mm	Separation of Centers mm
C	1 08	3 39
D	6 64	7 98

The double-star images to be studied were obtained by photographing test-objects C and D in a precision camera as in previous experiments The exposure times were 1^s, 2^s, 4^s, . . . 4^m in the case of object C, giving a series of double stars, the diameter of whose components varied regularly from 0 07 mm to 0 18 mm, varying somewhat with developer and development time. Many plates were exposed in this way and developed for different lengths of time in different developers in the hope of finding a variation in the effect which would lead to an understanding of its nature In this preliminary series of experiments no certain variation with developer and development time was disclosed In Table 30 giving the result of these measures, the mean result from all plates and all developers is given, eleven plates in all being measured

The fourth column of this table shows an undoubted repulsion of centers of such a magnitude as to completely overshadow the so-called gelatin and turbidity attraction effects

TABLE 30

RESULTS FROM DOUBLE STAR TEST-OBJECT C

Exposure Time	Separation of Edges	Diameter of Images	Separation of Centers	Computed Turbidity Correction
sec	mm	mm	mm	mm
1 0	0 096	0 076	0 1724	
1 9	084	088	1720	
3 8	071	101	1720	
7 5	060	112	1717	
15 0	048	124	1724	0 0000
30 0	036	136	1724	+ 0002
60 0	024	150	1749	+ 0008
120 0	016	160	1764	+ 0015
240 0	0 010	0 171	0 1813	+0 0026

The turbidity effect is given in the last column, computed from formula (66) with $\Delta = 12\ 5$

Test-object D —This test-object was designed for the purpose of finding out to what extent the repulsive effect shown in the case of test-object C depends on the *size* of the component images The measurements of only one plate are given in Table 31, as they are illustrative of the average result obtained Development was in pyro-soda The measurement in the case of test-object D was made in a different way from that of object C, for it can be objected that the result obtained for C was due to psychological causes, the bisection of centers in the process of measurement being influenced in a varying systematic manner by the presence of the neighboring or comparison

TABLE 31

RESULTS FROM DOUBLE-STAR TEST-OBJECT D

Exposure Time	Separation of Edges	Separation of Centers	Mean Longitudinal Diameter D_1	Mean Transverse Diameter D_2	D_2-D_1
sec	mm	mm	mm	mm	mm
1 0	0 068	0 413	0 346	0 350	0 004
1 9	059	413	354	358	004
3 7	052	412	360	368	008
7 5	049	416	367	376	009
15 0	044	419	374	387	013
30 0	041	422	380	396	016
60 0	035	425	389	402	013
120 0	032	428	396	411	015

image Accordingly, in the case of object D, the settings of the micrometer thread were made on the edges of each star, and the separation of centers was computed This also gives the longitudinal diameter of each stellar image As a check, the transverse diameter of each stellar image was measured, for a reason which will appear in Table 31

FIG 79
Images of a Double Star in Contact

The repulsion is seen to be greater than in the case of test-object C, although the minimum separation of edges is greater This was to be expected, if the explanation of the phenomenon presently to be given is the correct one In order to eliminate from D_2-D_1 any error in perfect roundness of the holes in the test-object, an auxiliary series of exposures was made in which each hole of the test-object was covered in turn The mean of a series of exposures made in this way gave D_2-D_1 $=0\ 004$ mm This correction has been applied in the foregoing table It is seen that as the images approach they become decidedly oval in form, the greater axis being transverse An enlargement of these images is shown in Fig 79, in which the oval form on the original negative was apparent to the eye

The Kostinsky Effect on Spectral Emission Lines —In order to investigate the Kostinsky effect on spectral emission lines, a test-object E was made having but one pair of slits of the following dimensions

Width of slits	0 36 mm
Distance between centers of slits	2 94 mm

On account of the importance of the Kostinsky effect on spectral lines, an extensive series of experiments was made with this test-object Five developers were used In all twenty plates were exposed, four for each developer The plates were measured both wet and dry In measuring the wet plates, pointings were made through the back, to eliminate capillarity error The exposures on each plate were timed to include images not far from threshold size at one end to images nearly in contact at the other end of the series

The results of Table 32 are plotted in Fig 80 In the first place it is noticed that the gelatin effect (wet-dry) is practically the same for all developers Since the minimum separation of

TABLE 32

RESULTS FROM TEST-OBJECT E (SPECTRAL DOUBLES)

Developers *a*, caustic hydroquinone *b*, pyro-metol, *c*, metol-hydroquinone, *d*, pyro, *e*, ferrous oxalate

Exposure Time	Separation of Centers					Separation of Adjacent Edges				
	a	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*
	Measured Dry									
	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
1 0 sec	0 1500	0 1480	0 1500	0 1498	0 1502	0 125	0 122	0 128	0 128	0 128
1 9	1490	1475	1490	1485	1498	116	112	120	119	120
3 7	1482	1472	1492	1478	1483	105	101	107	110	110
7 5	1478	1463	1488	1473	1465	094	091	095	098	099
15 0	1480	1450	1485	1470	1472	083	076	085	088	085
30 0	1478	1465	1472	1473	1478	072	065	072	076	073
60 0	1470	1445	1490	1458	1458	060	055	061	068	063
120 0	1470	1442	1468	1455	1433	047	C44	054	055	052
240 0	1470	1457	1482	1465	1438	037	033	044	048	042
480 0	0 1490	0 1470	0 1500	0 1492	0 1470	0 030	0 028	0 035	0 038	0 036
Mean	0 1481	0 1462	0 1487	0 1475	0 1470					
	Measured Wet									
1 0 sec	0 1510	0 1488	0 1502	0 1512	0 1512					
1 9	1515	1480	1510	1520	1512					
3 7	1510	1507	1522	1502	1493					
7 5	1498	1503	1525	1520	1498					
15 0	1512	1500	1522	1510	1505					
30 0	1515	1502	1540	1508	1512					
60 0	1522	1512	1543	1525	1507					
120 0	1520	1493	1533	1545	1502					
240 0	1555	1528	1545	1552	1515					
480 0	0 1560	0 1575	0 1562	0 1582	0 1538					
Mean	0 1522	0 1509	0 1530	0 1528	0 1509					
Mean Wet-Dry	0 0041	0 0047	0 0043	0 0053	0 0039					

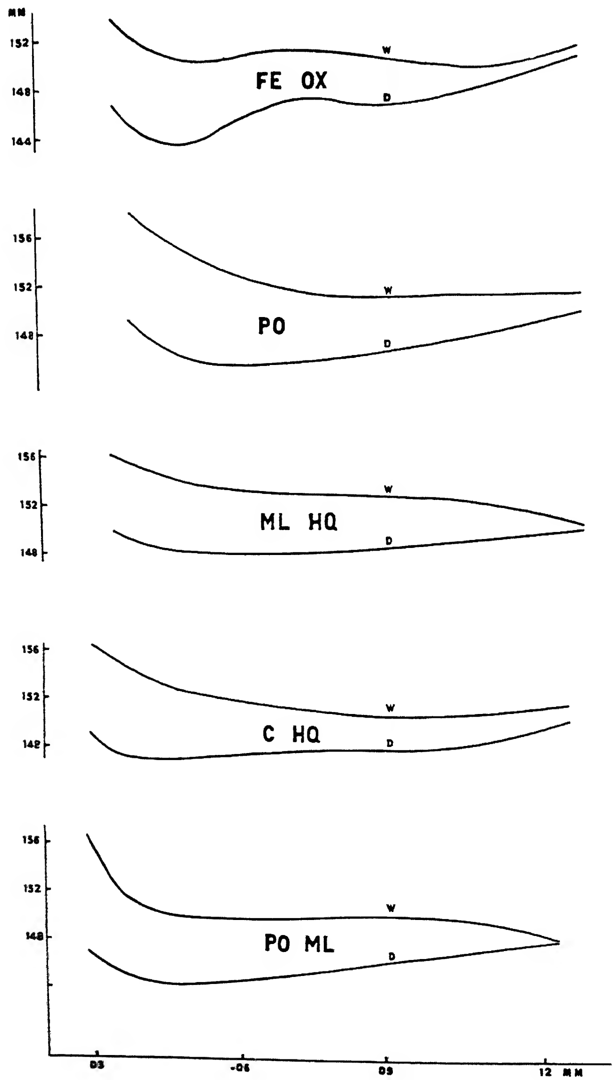


FIG 80
Separation of close emission-lines for various developers
Abscissa, edge separation

edges is 0 028 mm , the maximum turbidity effect is negligible (Table 25) Accordingly the curves (wet) give the true Kostinsky effect, it being assumed from previous work that the gelatin effect is not present in the wet condition of the plate It is possible that further work on the subject may modify this assumption

There appears from the curves to be considerable variation of the Kostinsky effect with developer The effect is greatest for pyro developer, and smallest for metol-hydroquinone The remarkable fact is brought out that in the case of measures on the *dry* plate, one to which naturally the greatest interest is attached, there is first an attraction, followed by a repulsion as the separation becomes smaller This may serve in part to explain the contradictory results obtained by various investigators The explanation is simple In the first stages the gelatin attraction is stronger than the Kostinsky repulsion, while in the latter stages, when the separation becomes less than about 0 06 mm , the Kostinsky repulsion predominates The behavior of ferrous oxalate is curious, and needs to be checked It is to be noted that in the case of all developers, development was fairly complete, having been pushed to the point where fog became noticeable

Another series of experiments is planned which will allow the curves to be followed closer to the point of contact In the present instance this could have been done simply by increasing the exposure, but the resultant lack of sharpness of the outline would have made accurate measurement difficult It is simply a matter of choosing test-objects of suitable dimensions, to get contact simultaneously with sharpness of outline

As to the cause of the Kostinsky effect, it seems reasonable to assume that it is a *developer* effect and but another manifestation of the Eberhard effect [1] In the process of development, reaction products are formed which are known to have a powerful restraining action on development The bright ring (Mackie line) surrounding sharp images is one manifestation of this action The action must be something more than a simple restraining one, for the bright ring is very often seen around minute stellar images The action must accordingly be a positive and permanent one, the products of development acting on the image in its immediate neighborhood as soon as formed, the effect spreading to adjacent regions as they diffuse, until their concentration becomes too low to be effective. With these assumptions, the Kostinsky "repulsion" of neighboring images is easily explained Consider a pair of adjacent

[1] Eberhard, E , l c.

images As development goes on, the liberated products of development pour into the intervening space from both sides, restraining or inhibiting development of the latent image in this region, as explained above But on the "far" sides the concentration of the products of development is only one-half Therefore, there is less restraining action of the latent image in these regions The net result of this non-symmetrical action on each image is an apparent repulsion or retrocession of centers This action is very well illustrated in Fig 79 which is an enlargement of a pair of close images, the oval form being quite perceptible In Fig 81 the action *downward* in the film is shown It will be noticed that the density or amount of silver is less in the center of the image than at the edge This is the true Eberhard effect, or dependence of *density* of image on its size The Kostinsky and Eberhard effects are considered to be but different manifestations of a common phenomenon in the Kostinsky effect we are concerned with the developer effect on size of image, in the Eberhard effect with the developer effect on density of image

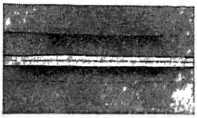


FIG 81
Two sections of photographic images showing Eberhard effect

Up to this point a seemingly confused mass of material has been presented, the results from which appear more or less contradictory in some cases the net result of an experiment is a repulsion, in others an attraction It is therefore pertinent to inquire if there is any principle governing the *net* effect It will be noticed that in general in those cases where a repulsion has been obtained, the images were formed by *overexposure,* and in those cases where an attraction was found, the exposures have been normal This is equally true with respect to the results obtained by other investigators quoted above, and those obtained by the writer as just described As to the cause, it is only necessary to assume that in over-exposure the retarding action of the products of development is very great, or that it is directly proportional to the exposure This is a not unreasonable assumption Instead of the separation of images, therefore, being the only controlling factor in the Kostinsky repulsion, or development effect, as has heretofore been assumed, an equally powerful factor is the exposure value

The conclusion is that in average work a slight *contraction* in separating distance of neighboring stellar images or bright emission lines in spectra is to be expected But on account of variations in exposure from normal to overexposure, it is by

no means certain in any given case that contraction will result there may be no effect, or there may even be a repulsion Summing up, if exposures are kept at a minimum, errors exceeding two or three microns need not be feared The importance of reducing the exposures to the lowest possible value is to be emphasized

Behavior of Close Absorption Lines—Upon applying the reasoning used in the treatment of close double stars and bright-line spectra or emission lines to the case of absorption lines, important differences arise in the case of all three of the factors of turbidity, gelatin effect, and development effect This is especially true in respect to the turbidity effect Consider two neighboring absorption lines, separated by an illuminated strip The intensity of illumination in this area must be less than in the regions adjacent to the "far" boundaries of the lines This becomes clear when it is noted that in the case of a turbid medium such as a photographic emulsion the intensity of the illumination at any point within it is the resultant of the flux at the surface combined with flux from the surrounding regions, which has been bent from its normal course by diffraction and reflection In the case in hand, the surrounding regions are the unilluminated absorption lines which lie on each side of the bright area There is therefore a deficiency of illumination at the adjacent edges of the absorption lines leading, in the developed image, to an apparent broadening of each line asymmetrically Owing to the factor of turbidity, therefore, but in quite a different manner from the action of turbidity on emission lines, the distribution of light leads to an apparent attraction of spectral absorption lines While the amount of the attraction in the case of emission lines can be calculated, as shown above, any simple mathematical treatment of the corresponding problem of absorption lines does not seem possible Only in a qualitative sense can the attraction be proved existent It is quite evident, moreover, that the action is greater the closer the lines, so that the general law of the action in both cases appears to be the same, although explained physically in different ways

Unlike the turbidity action, the disturbing action of the gelatin seems to be reversed in passing from emission to absorption lines Recalling that the movement of the edge of an image in the process of drying is always toward the region of greater density, and noting further that the weak density separating the absorption lines has less of a "pull" than the stronger density adjacent to the far edges, it follows that absorption lines are apparently repulsed owing to their asymmetric broad-

ening in drying It is quite likely that the action is weaker than in the case of bright-line spectra, for in order to preserve the definition and even the visibility of absorption lines the exposures given are relatively weak and the developed densities low It was shown on p 170 that the gelatin effect diminishes as the density of the image becomes less In bright-line spectra, on the contrary, the visibility is improved by long exposures, so that the central diffraction-image may be in this case very much overexposed The chemical actions in development are accordingly more intense, leading to greater disturbance than in the case of absorption lines

It remains to consider the development disturbance in close absorption lines Applying the hypothesis used in the previous cases of double stars and emission lines, namely, that the edge of a silver image loses density in development by an amount proportional to the chemical activity going on in the region back of the edge, it will be seen that the far edges of absorption lines suffer to a greater extent than the near or adjacent edges, so that here again an apparent repulsion takes place As in the case of the gelatin effect, the amount of the action should be less for absorption than for bright-line spectra, owing to the low densities concerned

In addition to the three disturbances just considered which are present in both emission and absorption spectra, there is a fourth disturbance which is effective only in the case of absorption lines On account of the great extent of the illuminated area of the plate back of the non-adjacent edges of the absorption lines, it is to be expected that the distribution or falling off in light-intensity at these edges is different from that at the near or contiguous edges From general considerations the light-curve at the far edges should be flatter, or κ, in the intensity equation (39), smaller Accordingly $\triangle$, the rate of growth of these edges with exposure, is greater (equation 51) The result is an apparent attraction of the lines The effect is enhanced by halation

Considering the totality of the disturbing actions concerned in the case of absorption lines, it may be predicted that errors in the measurement of adjacent absorption lines are small It is accordingly difficult to explain the difference between the Rowland and the Mount Wilson measures, found by St John, as being due to the phenomena here considered, which strengthens the case for the "personality" explanation of the differences, or at least a portion of them The anomalies found by Goos are more of the order that might be expected

The writer has made a series of laboratory experiments on artificial absorption lines In a first experiment a series of nine pairs of lines was arranged on the test-object (called F) to be photographed, the separation of the widest pair being 1 50 mm and of the closest 0 71 mm The width of the lines is

TABLE 33

ABSORPTION LINES, VARYING SEPARATION

Pair	Wet			Dry				Wet − Dry
	S_o	S_c	Diff	S_o	S_c	Diff	E	
	mm	mm	mm	mm	mm	mm	mm	mm
1	0 0793	(0 0793)	(0 0000)	0 0800	(0 0800)	(0 0000)	0 035	−0 0007
2	0703	0713	− 0010	0713	0719	− 0006	031	− 0010
3	0660	0669	− 0009	0673	0675	− 0002	029	− 0013
4	0557	0565	− 0008	0560	0570	− 0010	027	− 0003
5	0520	0531	− 0011	0527	0536	− 0009	024	− 0007
6	0487	0487	0000	0490	0491	− 0001	022	− 0003
7	0417	0435	− 0018	0420	0439	− 0019	021	− 0003
8	0410	0427	− 0017	0423	0431	− 0008	020	− 0013
9	0 0343	0 0374	−0 0031	0 0353	0 0378	−0 0025	0 012	−0 0010

equal to the separating spaces The distances on the test-object were accurately measured, so that, assuming no optical distortion, the relative correctness of the reduced separations in the photograph can be tested by simple proportion Table 33 shows the results obtained Each measured distance given is the mean of measurement of nine separate exposures

S_o is the measured separation of centers of the lines; S_c is the value computed as above described E is the separation of adjacent edges Results are given for plates measured both wet and dry The prediction that the actions would be small is verified The net result is an apparent *attraction* of close, spectral absorption lines Accordingly in this case the repulsive developer action must be relatively small.

It is important to obtain data on the behavior of absorption lines for overexposures, as has already been done for double stars and emission lines For this purpose two test-objects G and H were made, composed of parallel opaque strips about 6 mm wide In G the strips were placed far apart, in H close together The dimensions are as follows·

	G	H
	mm.	mm.
Separation of centers of strips .	8 694	6,6[illegible]
Separation of adjacent edges .	2 414	0 [illegible]

These were photographed in the camera as before, with [illegible] sure times from 1 to 120 seconds and completely de[illegible] in

caustic hydroquinone developer Two series of each were made and each plate measured twice, so that the numbers given below are the means of four separate values In order to eliminate personality error, settings were made on the four edges, from which the separation of centers was computed In the shortest exposure (1 sec) the densities were low and of the order obtained in practical work

TABLE 34

ABSORPTION LINES WITH INCREASINC EXPOSURE

Exposure Time	G			H			Width of Lines
	S Wet	S Dry	E	S Wet	S Dry	E	
sec	mm	mm	mm	mm	mm	mm	mm
1	0 453	0 457	0 125	0 347	0 350	0 008	0 34
2	453	456	129	346	351	013	34
4	449	455	136	345	349	018	33
7	449	458	149	341	347	024	32
15	451	457	161	337	345	036	31
30	448	454	175	333	341	047	30
60	449	454	190	333	341	060	28
120	0 448	0 455	0 188	0 331	0 337	0 070	0 27
Mean	0 450	0 456					

In the case of the wide pair G, no certain variation with exposure time is apparent But in the case of the close pair a strong attraction with increasing exposure, or else a repulsion with diminishing separations, is apparent In order to decide which of these alternative statements is the correct one, recourse may be had to the measured separations of the lines on the test-object itself, already given We find in this way

$$\text{Separation of H (wet)} = 0\ 453\ \frac{6\ 620}{8\ 694} = 0\ 345\text{mm}$$

$$\text{Separation of H (dry)} = 0\ 457\ \frac{6\ 620}{8\ 694} = 0\ 348\text{ mm}$$

Thus, except for a slight repulsive action, which can be neglected for the moment, the separations in the case of test-object H are correctly given for the short exposures even though the edges of the lines are nearly in contact In the case of the heavy exposures on H the lines are relatively far apart and yet a strong attraction has taken place, amounting

to 13μ. This is very readily explained as due to the fourth disturbing factor outlined above, namely, the greater turbidity on the far sides of the absorption lines combined with halation. In fact, the greater sharpness of the inside of each absorption line was apparent in the measurement. This cannot be considered an optical lack of sharpness since the field angle for the outside edge is only 6′.

A sufficient number of experiments have been described to show the great complexity of the mutual action of adjacent images. It has been the aim to cover the field only in a general way in order to disclose the factors of importance. A partial solution of the problem of eliminating mutual attractions and repulsions has been indicated. It remains for future work to make a detailed study of the action of various developers in the hope of securing one giving correct or normal development of the latent image.

CHAPTER VI

Film Distortion and Accuracy of Photographic Registration of Position

The subject of film distortion, by which is meant relative displacement of photographic images, due to any one of a number of possible causes, has occupied the attention of many investigators during the past twenty years The present chapter will be confined to the particular case of images relatively far apart or beyond the sphere of mutual influence It will be of interest to outline the most recent and important investigations and conclusions of a number of workers in this field There appears to have been but very little duplication of effort, each investigator treating the subject from an original and attractive viewpoint, so that the totality of results forms a well rounded whole The vicissitudes to which a photographic plate is exposed in the various operations of developing, fixing, and drying, combined with the very obvious discrete structure of the photographic image, a structure subject to the vagaries of grain clumping, leave a great deal to the imagination of the investigator in a serious questioning of the faithfulness of the photographic impression

The distortion of photographic film has been investigated by F Schlesinger[1] in 1906, in a manner which avoids several objectionable features of some previous investigations Two methods were used The distinctive feature of the first method consisted in a double development of the plate which had been exposed to a star field The plate being measured after each development by two observers, the distortions, if present, of either an accidental or systematic kind, can be differentiated by a simple mathematical process from the errors of bisection No systematic distortion was found The values of the distortion of an accidental nature, and the accidental errors of measurement were as follows

Distortion (mean error)	± 0009 mm.
Error of bisection of an image (mean error)	± 0020 mm

from which it is concluded that the plate distortions were small and less than the errors of measurement Nine subsequent prolonged soakings in water of the same plate failed to increase

[1] Schlesinger F On the distortion of photographic films Allegheny Obs Vol I No 1

this very minute distortion Concerning the conclusions to be derived from such a test, Schlesinger remarks "In the preceding experiments the first measurements were made from plates that had already gone through the processes of development There is a possibility that the film becomes set in some way in the drying after development so that thereafter it has a tenacity that it did not have before" To overcome this objection, a plate was spattered with ink, giving fine sharply outlined dots The coördinates of the dots were measured, after which the plate was immersed in a developing solution in which the developing agent itself was left out, then fixed washed and dried in the usual manner and remeasured The mean result from five plates was

Mean distortion error	± 0011 mm
Mean bisection error	± 0017 mm ,

agreeing with the result of the first experiment The author concludes that distortion errors are not to be feared

Important additions to the literature are contributed by C D Perrine[1] Perrine remarks "It has been the experience of practically all astronomers engaged in the measurement and reduction of photographs of stars or of spectra that the discordances found among such measures are much larger than can be accounted for by the errors of measurement alone" Perrine investigated the discordance or distortion of three kinds of plates, Seed 23, 27, and Seed Transparency, all coated on plate glass. Plates were exposed in the laboratory to a pattern plate containing small holes, as a check upon stellar exposure Distances of the order of 5 mm were measured His results are summarized in the following table

TABLE 35

Emulsion	Mean Ranges: Laboratory Exposures	Mean Ranges: Stellar Exposures
	mm.	mm.
Seed 23		0051
Seed 27	0051	.0043
Seed Transparency	0021	.0026

By range is meant the difference between the greatest and least result of measurement or the dispersion in readings. Perrine finds that the error of bisection forms but an insignificant part of the above, the range of the bisection error being 0.0006 mm. (mean of five settings, direct and reversed). The agreement between laboratory and telescope exposures is considered to

[1] Perrine C D Some results of a study of grains and structure of photographic film. Lick Obs Bulletin, No. 143. 1908.

place the burden of the difficulty on the photographic plate Accepting the result of numerous investigations, that distortion of the film itself is negligible, the author concludes that the difficulty is to be found in the images themselves, or, more specifically, in the structure of the image Examination under high magnification of a number of images, given identical exposures, discloses such diversity of outline that no doubt remains in the author's mind but that a true cause has been found He concludes "Broadly speaking, we may say that the trouble is due to a lack of homogeneity in the structure of the film If the grains of silver thrown down were much more uniformly distributed than at present, most of the trouble would probably vanish There are at least three possible causes for such an irregular structure as that observed It may result from a real difference in the sensitiveness of the silver grains, from the lack of uniform transparency of the gelatin covering, or from regions deficient in silver grains The volume of gelatin in a sensitive emulsion appears to be several times that of the silver grains [1] Hence it would seem that there must be considerable regions which are devoid of any silver grains an increase in the amount or a further subdivision of silver ought to reduce the scale of the lane structure in the penumbra of star images, with a consequent improvement in their forms and derived positions "

In a following paper[1] Perrine continues his investigations, studying in detail the effect of an increase in richness of silver in the emulsion and of a diminution in the size of the grain Seed X-ray plates were chosen on account of their greater richness in silver (about 25 per cent) In addition to ordinary development, the fine-grain development process of Lumière-Seyewetz was employed The following table summarizes the results obtained

TABLE 36

	Ranges	
Emulsion	Ordinary Development	Fine-Grain Development
Seed X-ray, ordinary glass	0030 mm	0029 mm
Seed 27, ordinary glass	(0064)	0040
Seed 27, plate glass	0044	0039
Seed 23, ordinary glass		0030
Seed 23, plate glass		0016
Seed Lantern, ordinary glass	0015	0018
Seed Transparency, plate glass	0022	0020

[1] About nine times in an average emulsion —F E R

[1] Perrine C D Results of some further studies on the structure of photographic films and the effect on measures of star images Lick Obs Bull No 178 1909

improvement in accuracy was found for fine-grain ent. It is considered that the expected advantage ease in richness of silver is borne out Further con- of importance deduced by the author are thin or ages show greater discordances than dense or fully ones, and that when the thickness of the emulsion is the discordances are also reduced In addition, pears to be a decided tendency for thick films images to give values of the distance systematically " He finds:

TABLE 37

late	Mean D
id (thick film)	17 4628 mm
w (thin film)	17 4600
tance on pattern plate	17 4604

hor concludes "Such an effect could be explained by ig a swelling action of some sort to take place after the were impressed upon the film, and during the process loping, fixing, washing and drying Just why a should take place is not clear It would seem more to expect a shrinkage, owing to the removal of un- silver from the film in fixing"

subject of film distortion has been investigated by S t[1], who covers important points in his investigation not red by previous workers To quote "The most im- features of the plan upon which my work was begun vestigations of the effects of (*a*) the position of the plate the processes of washing and drying, (*b*) the rate of (*c*) abrupt changes in the rate of drying during the , (*d*) changes in the position of the plate while drying; dness Emulsions on plate glass were also tried." sults were entirely negative The conclusions, however, importance and interest, which are summarized by the as follows: "(1) For the size of plate used (4 x 5) it was to be entirely indifferent whether the plate be vertical zontal during development, fixing, washing, and drying; thin the range of the observations, hardener, the rate of , and changes in the rate of drying and in the position plate during the process of drying introduced no general ion of the gelatin film; (3) local distortions were found ificial star plates and on spectrograms. These distor- were confined in each case to an area equal to a small on of a square millimeter The largest lateral displace-

recht, S. On the distortion of the photographic film on glass. Astrophys. J. 25 7

ment found at any point in the distorted area was 0 020 mm while the great majority were less than one-fourth of this amount, (4) The distortions appear to be principally of two different kinds one was due to an actual movement of a minute portion of the film, the other was an apparent shift of the image caused by the peculiar arrangement of the silver grains or to local differences in the sensitiveness of the film, (5) The results obtained from one plate-glass plate showed no advantages of the plate-glass over the ordinary commercial plates in the matter of distortions of the film"

The present writer[1], in 1912, uncovered an apparently new kind of film distortion Small plates, 27 x 37 mm , which were dried in a chimney type of drying box, showed a large general expansion amounting to one part in twelve hundred It was further found that if a plate containing star images, in this distorted condition, was soaked in water and dried in the ordinary way, the distortion disappeared In this same investigation a determination was made of the probable error of a single measured distance between réseau lines on plates dried in air and in alcohol, respectively There was a decided difference in favor of the alcohol-dried plates, as the following figures show

Probable error of a measured distance, air dried	$\pm$ 0 0020 mm.
Probable error of a measured distance, alcohol dried	0 0012 mm.

Uniformity of drying, secured very effectively by immersion in alcohol, is thus seen to be an important factor in reducing film distortion, at least on plates of these small dimensions

In Chapter V displacements in position of images which are close together are accounted for by assuming local inequalities in drying In the case of an isolated image such an hypothesis does not commend itself unless a macroscopic non-homogeneity of structure in the gelatin is assumed This would lead to local variations in the rate of drying Except for the observations of the writer quoted above, there are no well authenticated cases of general expansion or contraction of film on glass The amount of the observed displacements appears to be constant for all distances This fact suggests that the phenomenon of apparent displacements is of purely local origin and extent From this standpoint, namely, of

[1] Ross F E Latitude observations with photographic zenith tube at Gaithersburg Md Special Publ U S Coast and Geodetic Survey No 27, p 44

local action, collecting the various suggestions which have been advanced as possible causes, we have

1 Local drying strains,
2 Local variations in distribution of the silver bromide grains,
3 Local variations in sensitiveness,
4 Local variations in development,
5 Grain clumping, *i e*, graininess, or local variations in the distribution of the developed grains

It is quite evident that much more data must be accumulated before the phenomena in question can be established beyond dispute A number of experiments and measurements, suggested by various aspects of the subject, will be described In general, the distances measured were chosen small, which makes for greater accuracy, but not so small that the images might possibly be affected by mutual action Instead of forming artificial star images by contact printing, they were projected upon the plate in the precision camera It was felt that this is much the safer procedure The only drawback is a variation of focal distance from exposure to exposure owing to possible irregularities in the plate This effect should be small, however, since the plates are only one inch wide and are pressed firmly against a metallic bed in the plate holder In order to appraise the variation, if present, the pattern plate was made with four artificial stars distributed in a straight line If there is a variation of focus, the distance of the two outer stars should show a greater probable error It will be seen that this is not the case

Experiment A—Six plates were exposed in the precision camera (reducing 20 diameters) to the test-object containing four artificial stars, A, B, C, D, lying on a straight line. Ten exposures were made in rapid succession on each plate. The plate holder slides smoothly and accurately in metallic grooves. The plates were as follows

1 Fine-grain orthochromatic, thin emulsion (called astronomical);
2. Seed 30, medium thickness;
3. Triple coated orthochromatic.

These plates, of varying emulsion thickness, were chosen to test the thickness effect (see Perrine's result p 193) Two exposure times were chosen, giving weak or surface images, and

strong, deep images, respectively Each plate was measured 20 times, 10 direct and 10 reversed, and on 10 separate days "One measure" in Table 38 signifies the mean of a single direct and reversed measure, each image being bisected but once The readings were all started from the same point on the screw so that secular and periodic screw and bearing errors are eliminated The mean results are contained in the following table (the magnification used was 75 diameters)

TABLE 38

MEAN AND PROBABLE ERRORS

Plates	Emulsion Thickness	Diameter of Image	BC = Δ_1			AD = Δ_2		
			Mean Δ_1	Probable Errors		Mean Δ_2	Probable Errors	
				One Distance (Photographic)	One Measure (Personal)		One Distance (Photographic)	One Measure (Personal)
Weak (2^s) exposure	mm	mm	mm	mm	mm	mm	mm	mm
Astronomical	014	021	1 3321	00040	00043	2 6512	00085	00058
Seed 30	030	027	1 3317	00075	00063	2 6519	00134	00055
Triple C Ortho	060	024	1 3318	00084	00065	2 6520	00180	00068
Means			1 3319	00066	00057	2 6517	00133	00060
Heavy (60^s) exposure								
Astronomical	014	061	1 3319	00046	00043	2 6510	00064	00042
Seed 30	030	067	1 3314	00072	00054	2 6513	00072	00053
Triple C Ortho	060	065	1 3324	00075	00062	2 6519	00080	00049
Means			1 3319	00064	00053	2 6514	00072	00048

Comparing in this table the means of mean Δ for weak and heavy exposures, it is found that

$$\text{Mean } \Delta \text{ (heavy exposure)} - \text{Mean } \Delta \text{ (weak exposure)} = 0\ 00015 \text{ mm}$$

Comparing mean Δ by plates, agreement is not so marked However, the pronounced increase in Δ for thick emulsions found by Perrine is not in evidence

Comparing probable errors, the superiority of the thin fine-grain astronomical emulsion is manifest This plate is by no means slow It accordingly has distinct advantages in as-

ronomical work The photographic probable errors are in general smaller than obtained by previous workers In fact hey are so small as to materially increase faith in the accuracy of photographic registration of position, under properly controlled conditions There is, however, no doubt but that occasional apparent displacements of images manifest themselves, amounting to 0 002 mm at the maximum In order to see if there is any dependence of displacement on the character of the image, all images on the above series of plates concerned n producing large displacements were examined In every case the images of each pair were round and regular, showing no sign of disturbance On the other hand, distances measured between images which were very irregular in outline—cases where one would expect an apparent displacement—did not indicate anything abnormal These facts tend to throw doubt on the irregularity-of-outline theory of image displacements and are accordingly more favorable to the theory of local gelatin disturbance, *i e*, disturbances confined to minute volumes of gelatin such as would be produced, for example, if the gelatin were not strictly homogeneous The comparatively large value of the photographic error for Δ in the case of weak exposures, shown in Table 38, is difficult to explain That it cannot be caused by irregularities in focal distance mentioned above is indicated by the following results

TABLE 39

Δ FOR WEAK EXPOSURES

	(Triple-Coated Ortho Plate)			
Exposure	Δ_1	v_1	Δ_2	v_2
1	1 3331	−13	2 6492	+28
2	3319	− 1	6478	+44
3	3325	− 7	.6536	−16
4	3311	+ 7	6549	−29
5	3329	−11	6553	−33
6	3295	+23	6527	− 7
7	3329	−11	6520	0
8	3323	− 5	6503	+17
9	3310	+ 8	.6506	+14
10	3309	+ 9	6540	−20
Means	1.3318		2 6520	....

Each Δ is the mean of 20 measures on 10 days, so that accidental measuring errors are eliminated Comparing corresponding v's there is no evidence of correlation

The fidelity of the photographic plate in recording distance is primarily dependent upon the physical properties of gelatin, which are peculiar and exceedingly complex. The present form of gelatin is sensitive both to heredity, history, and immediate environment. "Any 'structure' is not inherent in the gelatin but is an environment impress, a strain structure in the original mass."[1] It is to be pointed out that the form characteristics are not to be regarded as merely two-phase, the wet and dry phase, with intermediate values, for even in the dried condition, two or more dimensional forms can exist in the same mass, depending on its previous history. An example of this is recorded on p. 194. Numberless experiments suggest themselves which would be useful in the study of the forms of gelatin masses in general, and in particular, in the study of sheets of gelatin containing emulsions coated on plates. Sheppard and Elliott (*l. c.*) have made a partial study of the shapes which drying masses of gelatin take under various conditions, in particular with one face constrained, which is the case of interest in photography. From their experiments it might be inferred that the upper layer of gelatin on a photographic plate *contracts* on drying, at least at the edges of the plate. It is of interest to make a special study of what happens in this particular and important case. It may not be of practical interest to the astronomer to know what becomes of the photographic image when the film is wet. Knowing that the images on hydration and dehydration move in a general direction perpendicular to the plate, it is only necessary that these opposite movements take place along the same path. This is, of course, an assumption which must be proved. The expansion of the film in development takes place under entirely different physical and chemical conditions than its contraction on drying, because of the removal of the unexposed silver bromide, and other factors. The natural inference is that distortions would be very likely to take place. This has been the viewpoint of astronomers who have gone to considerable trouble to determine their character and the amount of the distortions.

Measurements on Wet and Dry Plates.—In order to observe the effect of swelling, moderately large distances were chosen. Since the effect, if any, should be a maximum on thick films, a triple-coated plate was used. Two groups of images, 30 mm. apart, symmetrically located on a plate 1 x 5 in., were measured. The images were small and confined to the surface so that any surface creep would be disclosed. The fixing bath contained no hardener. After measurement, dry and wet, the

[1] Sheppard, S. E. and Elliott, F. A. The drying and swelling of gelatin. J. Amer. Chem. Soc. **44**, 373, 1922.

plate was bathed in a hardening (formalin) bath and the measurement repeated △ below is the mean of four independent distances, measurements being made direct and reversed

	Mean △
Plate measured wet	30 1380 mm
Plate measured dry	30 1380
After formalin bath, wet	30 1368
After formalin bath, dry	30 1389

These measures show that even under the extreme conditions chosen, the movements of the images in swelling and in drying are outward and inward along the same path, which path is without doubt perpendicular to the plate A very minute effect is shown to have resulted from the hardening bath but is so small as to be doubtful The effect of hardening is taken up in more detail on p 201

It is well known that in the case where images are located near the edge of a plate, measures are unreliable In order to find out the exact nature of the phenomenon exhibited in this case, a very narrow strip, 4 3 mm wide, was cut from a plate containing exposures of the four-hole test-object in such a way that the two outer images, *A* and *D* were but 0.8 mm from the edge This plate was then subjected to a series of hydrations and dehydrations and measured at each phase The following table contains the results Soakings were for 30 minutes in water at 70° F

TABLE 40

Distortions Near Edge of Plate

		△			
	Original	*AB*	Increase (μ)	*BC*	Increase (μ)
1	Dry	2 655		1 333	
2	Wet	2 665	+10	1 337	+ 4
3	Dry	2 702	+47	1 372	+39
4	Wet	2 680	+25	1 345	+12
5	Dry	2 707	+52	1 368	+35
6	Wet	2 685	+30	1 352	+19
7	Dry	2 712	+57	1.374	+41
Distance of images from edge			0 8mm		1 5 mm.

Very pronounced distortions are exhibited in this table which are seen to be considerably less in the case of the images *BC* further from the edge. It is to be noted that after the first wetting the film always *expands* on drying

Further data were obtained from similar measurements on a wider plate (8 1 mm wide) The plate was alternately wet and dried seven times The mean results are as follows

TABLE 41

	AD	BC
Original, Dry	2 653 mm	1 331 mm
Measured Wet	2 653	1 333
Measured Dry	2 659	1 336
Distance of images from edge	2 7	3 4

The same phenomenon is exhibited, but to a lesser extent, as was to be expected, since the distances from the edge are greater

Cause of Edge Distortion—At first sight, it would appear that on drying there must be a shrinkage of the gelatin at the edge of an image, taking place in the plane of the plate, following the analogy of a cube of gelatin drying with one face constrained In the above experiments, however, the opposite was found to be the case, *i e*, the gelatin *expanded* on drying The explanation of the phenomenon appears to be the same as the one proposed in Chapter V, in explaining the mutual action of adjacent images It is imagined that in drying, whenever any differential action occurs such as one portion of the plate drying more quickly than another, there is a *migration* of gelatin with its encompassed images which takes place in the direction of the region which has dried first H Stintzing[1] finds that it is a general property of colloid systems, of which gelatin is an example, that exposure to radiation produces an increase in concentration of the colloid in the portions insolated, migration taking place from the unaffected regions The phenomenon takes place only if evaporation is permitted The author explains the phenomenon as a change in the distribution of the colloid resulting from local inequalities in the rate of evaporation and affirms that this is a general property of colloid systems

This principle of migration is manifestly applicable to the edges of plates which dry more quickly than the center The drying line accordingly creeps in from edge to center, accompanied by migration of gelatin outward in accordance with this principle. It will be convenient to use the term "hydration gradient" which may be defined as the maximum derivative of the specific water content with respect to any direction

[1] Stintzing, H Der Einfluss des Lichtes auf Kolloide Systeme Kolloidchem Beih 6 231 1914

in the plane of the plate On account of the rapid setting of the gelatin at the extreme edge, leading to a very large hydration gradient, there is greater differential action in this region and accordingly a greater transfer of gelatin The hydration gradient at any distance from the edge, greater than 10 mm roughly, appears to be so low that transfer of gelatin and consequent distortion are insignificant In this case the movements are strictly up and down or normal to the plate. It is quite probable that irregularities in the thickness of the film-coating lead to uneven drying and consequently to distortions.

Effect of Hardener —A small plate 16 x 24 mm was cut in such a way that a group of images lay near each long edge in a line perpendicular to it After measuring dry and wet, the plate was bathed in formalin and remeasured. Table 42 contains the results In the case of Δ_1, the images are 1.3 mm. from the edge of the plate

TABLE 42

Measures on Hardened Plate

	Δ_1		Δ_2		Δ_3		Δ_4	
	mm	Exp μ	mm	Exp μ	mm	Exp. μ	mm.	Exp. μ
Original (after cutting dry	14 173		12 863		10 194		8.881	
Soaked 1 hour in water, wet	14 174		12 863		10 193		8 877	
Soaked 1 hour in water dry	14 195	+22	12 881	+18	10 204	+10	[illegible]	+8
Soaked 1 hour in water then 15 minutes in formalin wet	14 180	+ 7	12 866	+ 3	10 195	+ 1	[illegible]	− 1
Soaked 1 hour in water then 15 minutes in formalin, dry	14 194	+21	12 871	+ 8	10 199	+ 5	[illegible]	− 1
Soaked 1 hour in water then 15 minutes in formalin wet	14 193	+20	12 872	+ 9	10 201	+ 7	[illegible]	− 1
Soaked 1 hour in water then 15 minutes in formalin, dry	14 191	+18	12 872	+ 9	10 200	+ 6	[illegible]	− 0
Soaked 1 hour in water then 15 minutes in formalin wet	14 190	+17	12 870	+ 7	10.200	+ 6	[illegible]	+1
Distance of each image from edge of plate	1 3		2 0		3.8		[illegible]	

It will be noticed from Table 42 that no effect results from the hardening bath until the plate has dried. The small effect appearing in the fourth line is doubtless due to partial [illegible] before the measures could be [illegible]. After [illegible] before the measures could be [illegible] obtained on the plate [illegible] true distances can no longer be

soaking in water Distances are now the same, whether measured wet or dry and correspond to the false distances indicated in the third line of the table and not to the true distances of the first line The reason for this is not far to seek The hardening has destroyed nearly all the power of the gelatin to swell on immersion in water In the case of the above plate, the swelling was found to be only 40 per cent as compared with an average of 700 per cent on unhardened plates

Effect of Intensification —Although in these experiments no clear effect upon position measurements is disclosed which is due to the raggedness of outline of the images measured, there remains the possibility of such errors being present The *a priori* reasons for the existence of such are strong If actually present there is the attractive probability that intensification of the images will lead to improved results Such improvement should be especially noticeable in the case of weak images in which the defects of structure are more pronounced Fig 82 shows a number of photomicrographs of weak images In the second line of the figure the same images are shown after intensification Images having an unusual amount of distortion were chosen One is of an equilateral triangular form It is to be understood that these irregularities are not due to the optical system but are accidental configurations on the photo-

TABLE 43

Effect of Intensification

Exposure	*AD*				*BC*			
	Before Intensification		After Intensification		Before Intensification		After Intensification	
	Δ_1 mm	v	Δ_1 mm	v	Δ_2 mm	v	Δ_2 mm	v
1	2 6505	+14	2 6508	+ 5	1 3297	+20	1 3304	+14
2	6526	− 7	6518	− 5	3315	+ 2	3305	+13
3	6539	−20	6530	−17	3308	+ 9	3302	+16
4	6539	−20	6525	−12	3329	−12	3328	−10
5	6524	− 5	6521	− 8	3314	+ 3	3317	+ 1
6	6540	−21	6526	−13	3318	− 1	3326	− 8
7	6502	+17	6495	+18	3324	− 7	3318	0
8	6483	+36	6480	+33	3327	−10	3326	− 8
9	6510	+ 9	6496	+17	3310	+ 7	3310	+ 8
10	6525	− 6	6533	−20	3332	−15	3347	−29
Means	2 6519		2 6513		1 3317		1 3318	

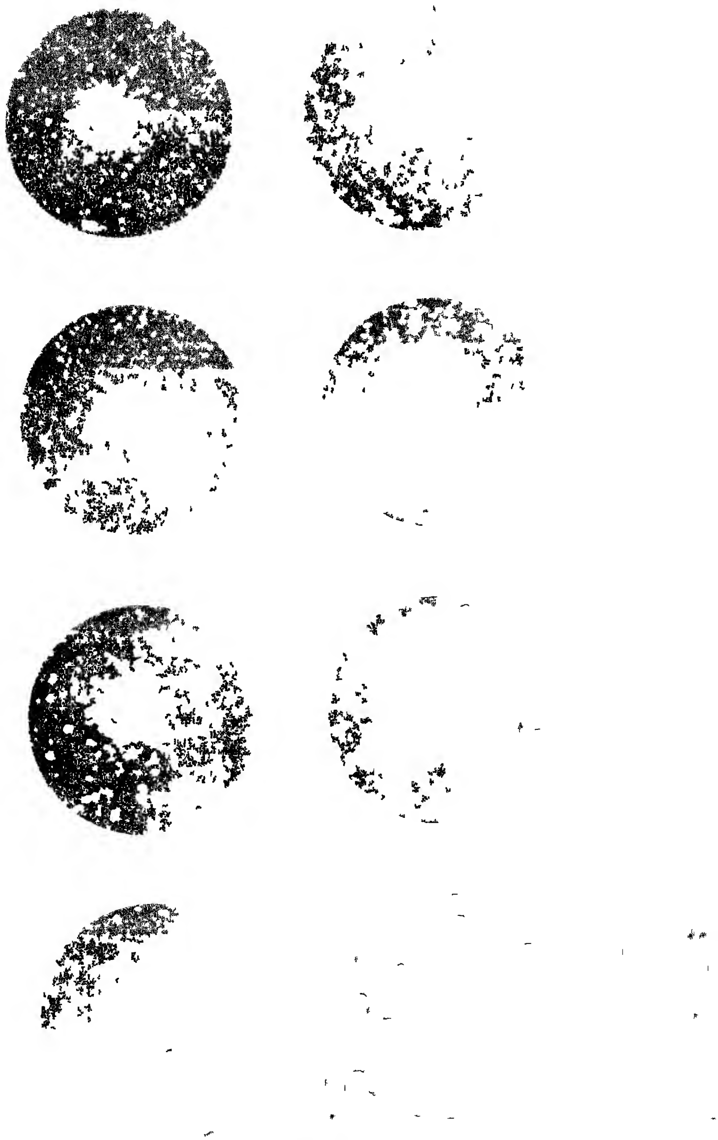


FIG 82
Photomicrograms of weak star images before and after intensification

graphic plate It will be noted that intensification has not altered the larger peculiarities In the case of the smaller defects which include open spaces, bays and capes, there is a decided filling in so that in appearance, at least, there is considerable improvement on intensification

In order to make a numerical test of the effect of intensification, a Seed 30 plate containing weak exposures was chosen The system of exposure has been described on p 196 In fact, the plate chosen figures in the second line of Table 38 The following table contains the results of measurement of this plate before and after intensification Each Δ is the mean of ten measurements made on as many days

Comparison of the *v*'s before and after intensification, in the above table, proves that intensification has had no effect on the apparent occasional displacements of star images, which are indicated by abnormal values of *v* On account of the improvement in the appearance of intensified images noted above, it might be expected that the accidental measuring errors are reduced That this is the case is shown by the following

TABLE 44

Probable Error of a Single Measurement (Mean of Direct and Reversed)

	Δ_1 mm	Δ_2 mm
Before Intensification	± 00055	± 00063
After Intensification	00044	00050

Accordingly, the only gain on intensification appears to be a diminution in the error of measurement The constant errors in position of the image are not affected

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Index of Authors

Index of Subjects